CERN-TH/99-281
September 1999
hep-ph/9910534

Top quark production near threshold*

M. Beneke[a]

[a]Theory Division, CERN, CH-1211 Geneva 23, Switzerland

The present theoretical status of top quark pair production near threshold at (future) e^+e^- ($\mu^+\mu^-$) colliders is summarized.

1. INTRODUCTION

Only a few of the top quark's properties are currently known. In the future the Tevatron and the LHC, and a Linear Collider, will measure many of its couplings in great detail. This will tell us whether the top quark is just another standard model quark (in which case it would be a rather uninteresting one) or whether it plays a special role in physics beyond the standard model, as perhaps its large mass suggests.

The measurement of the top quark mass and decay width through a scan of the top cross section near the pair production threshold is unique to a high-energy lepton collider – and only indirectly related to non-standard model physics. (At a hadron collider the top quark pair invariant mass distribution is analogous to the threshold scan. However, the invariant mass can probably not be measured with an accuracy below 1 GeV as is necessary to make the subsequent discussion relevant.) Besides providing an accurate measurement of these parameters, the threshold scan probes a new and interesting regime of strong interaction dynamics: for a very short time between its birth and death through single top quark decay, the $t\bar{t}$ pair is the only strongly interacting system we know which is bound by a perturbatively calculable heavy quark potential. Contrary to charmonium and bottomonium, which are partly non-perturbative, toponium properties can be computed systematically.

Although leading order and next-to-leading order calculations of the threshold production cross section have been completed quite some time ago [1], the methods to carry out such calculations systematically have been fully developed only recently. They have been used so far to compute the $t\bar{t}$ production cross section to next-to-next-to-leading order (NNLO) in the threshold region [2–4]. In this talk, based on [3], I give a brief account of the calculational tools involved and discuss the surprising conclusions that emerged from the NNLO calculation.

2. SYSTEMATICS

The threshold region is defined by the scaling rule $v \sim \alpha_s(m_t v)$, where v is defined through $E \equiv m_t v^2 \equiv \sqrt{s} - 2m_t$ and $\sqrt{s} = q^2$ is the centre-of-mass energy squared. m_t denotes the pole mass – mass renormalization is an important issue, which will be discussed in detail later. Since $v \ll 1$, several scales are relevant to $t\bar{t}$ production near the threshold. The approach described below treats these scales sequentially, working downwards from the largest scale m_t. To do this, one has to know what the relevant momentum regions are. Just as one distinguishes hard, collinear and soft particles in high-energy collisions of (massless) quarks and gluons, one can identify the following momentum regions relevant to non-relativistic heavy quarks [5]: hard (energy $l_0 \sim m_t$, momentum $\boldsymbol{l} \sim m_t$ – referring to a frame where $\boldsymbol{q} = 0$) heavy quarks and gluons (light quarks), soft ($l_0 \sim m_t v$, $\boldsymbol{l} \sim m_t v$) heavy quarks and gluons, potential ($l_0 \sim m_t v^2$, $\boldsymbol{l} \sim m_t v$) heavy quarks and gluons and ultrasoft ($l_0 \sim m_t v^2$, $\boldsymbol{l} \sim m_t v^2$) gluons. For top quarks the

*Talk presented at the High Energy Physics International Euroconference on Quantum Chromodynamics (QCD'99), Montpellier, France, 7-13 July 1999.

soft scale turns out to be of order 20 GeV, the ultrasoft scale of order 2 GeV – large enough for a perturbative treatment. However, the presence of the small kinematic parameter v implies that some terms have to be summed to all orders in α_s.

2.1. Coulomb resummation

The exchange of a Coulomb (potential) gluon results in a 'correction' of order $\alpha_s/v \sim 1$. The leading order Coulomb interaction must be treated non-perturbatively. Defining the R-ratio for the top production cross section through a virtual photon in the usual way, this leads to a succession of LO, NLO, ... approximations defined by keeping all terms of the form

$$R \equiv \sigma_{t\bar{t}}/\sigma_{\mu^+\mu^-} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \cdot \big\{1\,(\text{LO}); \alpha_s, v\,(\text{NLO}); \alpha_s^2, \alpha_s v, v^2\,(\text{NNLO}); \ldots\big\}. \quad (1)$$

Note that at NNLO we do not need the exact perturbative coefficient of the α_s-expansion, but only the first three terms in an expansion in v. This makes it useful to construct an expansion method (the 'threshold expansion') which allows us to compute the expansion coefficients without knowing the exact result [5].

2.2. Logarithms

The resummation scheme (1) replaces the conventional α_s-expansion. Each further order in the resummation improves the theoretical accuracy by one power of α_s as usual. But because of the very different momentum scales involved, large logarithms of v and $v^2 \sim 1/100$ are left over. These logarithms can also be summed using renormalization group methods. The leading logarithmic (LL), next-to-leading logarithmic (NLL), ... approximation is defined by keeping all terms of the form

$$R = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_l (\alpha_s \ln v)^l \cdot \big\{1\,(\text{LL}); \alpha_s, v\,(\text{NLL}); \alpha_s^2, \alpha_s v, v^2\,(\text{NNLL}); \ldots\big\}. \quad (2)$$

In practice, the summation of logarithms is done first by summing them into the coefficient functions of operators in the effective field theories discussed below. The summation of Coulomb α_s/v corrections is performed at the end by computing scattering amplitudes in the effective theory perturbatively.

2.3. Treatment of the top quark width

The decay $t \to Wb$ proceeds rapidly and the single top decay width cannot be neglected near threshold. In fact, in the standard model, $\Gamma_t \sim m_t v^2$. For hard and soft top quarks the width is a small correction to the quark propagator and can be expanded. For potential top quarks with energy $E \sim m_t v^2$ and momentum $\boldsymbol{p} \sim m_t v$, we can approximate the quark propagator

$$\frac{1}{\not{P}_t - m_t - \Sigma(P_t)} \approx \frac{1}{E + i\Gamma_t - \boldsymbol{p}^2/(2m_t)}\,[1 + \mathcal{O}(v)]\,. \quad (3)$$

The width is a leading order effect (which cannot be expanded), but can be taken into account at leading order by making the top quark energy complex: $E \to E + i\Gamma_t$, where Γ_t is the gauge-independent on-shell decay width. Beyond leading order, counting $\Gamma_t \sim m_t v^2$, the self-energy has to be matched to better accuracy. The correction terms relate to the off-shell self-energy and carry electro-weak gauge-dependence. A complete NNLO result in the presence of a width that scales as above therefore includes electroweak vertex corrections as well as single resonant backgrounds and non-factorizable corrections to the physical $WWb\bar{b}$ final state. A systematic treatment of these complications has not been attempted yet.

2.4. Non-perturbative effects

On top of the resummed perturbative expansion there exist non-perturbative effects suppressed by powers of the QCD scale Λ. The leading non-perturbative correction to the $t\bar{t}$ cross section far from threshold is supposed to be described by the gluon condensate and scales as $(\Lambda/m_t)^4$. As $m_t v^2 \gg \Lambda$, the operator product expansion remains valid near threshold, but the relevant scale is now $m_t v^2$ rather than m_t. Accounting for the velocity suppression of the top coupling to a dynamical gluon, we obtain the estimate $\delta R_{\text{NP}}/R \sim v^2(\Lambda/(m_t v^2))^4$, which is very small indeed. It therefore seems justified to neglect non-perturbative effects, although a more detailed investigation would certainly be worthwhile.

Note that there is no room for phenomenological, non-perturbative modifications of the heavy quark potential, such as adding a linear term of order $\Lambda^2 r$, in the present approach. Since $m_t v^2 \gg \Lambda$ there is no reason to assume that a non-perturbative gluon with momentum of order Λ would give rise to an instantaneous interaction. Rather it would modify the propagation of ultrasoft gluons in the effective theory discussed below, while leaving the potential unmodified. Estimating the size of non-perturbative effects by adding $\Lambda^2 r$ to the potential would result in the over-estimate $\delta R_{\rm NP}/R \sim (\Lambda/(m_t v))^2$.

2.5. Present status

NNLO calculations have now been done by several groups [2–4]. It is known that leading logarithmic contributions are absent [3]. Some NLL and NNLL corrections are known, but a systematic implementation of the renormalization group is still missing. (As correctly pointed out in [6], the summation of NLL effects in [3] is not complete, contrary to the statement made there.) There is a calculation of some potentially important NNNLO terms [7]. Finite width effects have been treated only to *leading* order, i.e. using $E \to E + i\Gamma_t$ (as in [1]), or variants that have equivalent parametric accuracy. Some non-factorizable contributions have been studied near threshold [8]. As already mentioned, non-perturbative corrections have not yet been estimated, but they are expected to be small.

In order to optimize an accurate top quark mass determination it is important to choose an appropriate mass renormalization scheme, different from the on-shell scheme [9] that has been used by convention until recently. The latest calculations [3,4] make use of such schemes, though different ones.

3. CALCULATION IN THREE STEPS

We sketch how the resummed threshold cross section is obtained. This is done by constructing, in two steps, an effective theory in which all modes except potential quarks and ultrasoft gluons are integrated out. The last step implies computing the cross section perturbatively in the effective theory. Although the cross section is ultraviolet and infrared finite, intermediate steps in this construction lead to infrared and ultraviolet divergences, which have to be regularized consistently. It is convenient and simple to use dimensional regularization.

3.1. Step 1: Matching on NRQCD

We begin with integrating out hard (relativistic) quarks and gluons. Their contribution goes into the coefficient functions of the non-relativistic QCD (NRQCD) Lagrangian [10] and into the coefficient functions of non-relativistic external currents. The effective Lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm NRQCD} &= \psi^\dagger \left(iD^0 + \frac{\boldsymbol{D}^2}{2m_t} + i\Gamma_t\right)\psi \\ &\quad + \text{NNLO} + \text{antiquark terms} + \mathcal{L}_{\rm light}, \qquad (4)\end{aligned}$$

where ψ is the non-relativistic top quark field. Only the terms that contribute at leading order are written out explicitly. (The remaining ones can be found in [3].) The bilinear term proportional to Γ_t arises from matching the heavy quark self-energy to leading order and effects the substitution $E \to E + i\Gamma_t$ as discussed above. The leading order terms are not renormalized. Hence logarithms can appear only in the NNLO terms.

The top quark vector coupling to the virtual photon in the effective theory is given by

$$\bar{t}\gamma^i t = c_1\, \psi^\dagger \sigma^i \chi - \frac{c_2}{6m_t^2}\, \psi^\dagger \sigma^i (i\boldsymbol{D})^2 \chi + \ldots, \qquad (5)$$

where the ellipsis refers to terms beyond NNLO and χ denotes the top anti-quark field. At NNLO, we can use $c_2 = 1$, while c_1 is needed at order α_s^2. The two-loop contribution to c_1 has been computed in [11]. To leading-logarithmic, NLL, NNLL, ... accuracy one also needs the 1-loop, 2-loop, 3-loop, ... anomalous dimension of the operator $\psi^\dagger\sigma^i\chi$. The 1-loop anomalous dimension vanishes, so there are no LL effects. The 2-loop anomalous dimension is known [11] and contributes NLL terms. In addition the NNLO couplings in (4) mix into $\psi^\dagger\sigma^i\chi$ through ultraviolet divergent potential loops. Hence the LL renormalization of these couplings contributes NLL terms to the $t\bar{t}$ cross section.

3.2. Step 2: Matching on PNRQCD

The loop integrals constructed with the non-relativistic Lagrangian still contain soft, potential and ultrasoft modes. Near threshold, where energies are of order $m_t v^2$, only potential top quarks and ultrasoft gluons (light quarks) can appear as external lines of a physical scattering amplitude. Hence, we integrate out soft gluons and quarks and potential gluons (light quarks) and construct the effective Lagrangian for the potential top quarks and ultrasoft gluons (light quarks). Because the modes that are integrated out have large energy but not large momentum compared to the modes we keep, the resulting Lagrangian contains instantaneous, but spatially non-local interactions in addition to the local interactions inherited from the NRQCD Lagrangian. In the simplest case, the instantaneous interactions reduce to what is commonly called the 'heavy quark potential'. This theory is appropriately termed potential non-relativistic QCD (PNRQCD) [12]. Although both are integrated out together, it is necessary to distinguish potential gluons (which are off mass shell) and soft gluons (which are on-shell) to obtain an homogeneous (single scale) expansion near threshold [5]. However, both contribute to the heavy quark potential and only their sum is gauge-invariant (see also the discussion in [13]).

There is no further modification of the external vector current induced by matching on PNRQCD up to NNLO. For the effective Lagrangian we quote again only the leading order Lagrangian explicitly:

$$\begin{aligned}\mathcal{L}_{\rm PNRQCD} &= \psi^\dagger \left(i\partial^0 + \frac{\boldsymbol{\partial}^2}{2m_t} + i\Gamma_t\right)\psi \\ &+\chi^\dagger \left(i\partial^0 - \frac{\boldsymbol{\partial}^2}{2m_t} + i\Gamma_t\right)\chi \\ &+\int d^{d-1}r\,\left[\psi^\dagger\psi\right](r)\left(-\frac{C_F\alpha_s}{r}\right)\left[\chi^\dagger\chi\right](0) \\ &+\text{NLO}. \end{aligned} \qquad (6)$$

The leading order Coulomb potential is part of the leading order Lagrangian and cannot be treated as a perturbative interaction term as anticipated above. At NNLO the two-loop correction to the Coulomb potential [14] is required, as well as the potentials of the form α_s^2/r^2, α_s/r^3 and $\alpha_s\delta(r)$ and kinetic energy corrections. A remarkable feature is that ultrasoft gluon interactions contribute only from NNNLO on. There are no gluon (light quark) fields in the PNRQCD Lagrangian to NNLO. It is after matching NRQCD on PNRQCD that the computation of the threshold cross section turns into an essentially quantum-mechanical problem. Nevertheless, matching is crucial. The non-Coulomb potentials result in ultraviolet divergent integrals which we regulate dimensionally (the potentials are therefore needed in d dimensions – see [3]). The resulting regulator-dependence cancels with the regulator-dependence of the matching coefficients in NRQCD and PNRQCD. (This provides a check of the logarithm in the 2-loop contribution to c_1 and verifies the 2-loop anomalous dimension of the non-relativistic current computed in [11] from the effective theory side.)

The potentials should be considered as short-distance coefficients of non-local, instantaneous operators in the PNRQCD Lagrangian. This implies that the potentials are infrared finite, but can contain (parametrically large) logarithms $\ln(\mu_s/\mu_{us})$, where $\mu_s \sim m_t v$ is the scale at which NRQCD is matched on PNRQCD and $\mu_{us} \sim m_t v^2$ is the renormalization scale of PNRQCD. Explicit calculation of the potentials to NNLO demonstrates that they do not contribute LL terms. An example of a NNLL term is provided by the logarithm in the Coulomb potential at order α_s^4 [15].

3.3. Step 3: PNRQCD perturbation theory

Perturbation theory in PNRQCD has much in common with perturbation theory in quantum mechanics. The lowest order Coulomb potential is part of the unperturbed Lagrangian. Perturbation theory in PNRQCD does not use ordinary free (non-relativistic) quark propagators, but a propagator for a $t\bar{t}$ pair in the presence of the Coulomb interaction. The corresponding

Coulomb Green function $G_c(\boldsymbol{r},\boldsymbol{r}';\bar{E})$ solves

$$\left[-\frac{\boldsymbol{\nabla}_{\boldsymbol{r}}^2}{m_t} + V(\boldsymbol{r}) - \bar{E}\right] G_c(\boldsymbol{r},\boldsymbol{r}';\bar{E}) = \delta^{(3)}(\boldsymbol{r}-\boldsymbol{r}'), \tag{7}$$

where $\bar{E} = E + i\Gamma_t$ and $V(\boldsymbol{r}) = -C_F\alpha_s/r$. (More precisely, the dimensionally regularized Coulomb Green function is defined through the corresponding integral equation in momentum space.) The non-leading potentials can be treated perturbatively, which leads to integrals of the form

$$\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{d^3\boldsymbol{q}_1}{(2\pi)^3}\frac{d^3\boldsymbol{q}_2}{(2\pi)^3}\,\tilde{G}_c(\boldsymbol{p},\boldsymbol{q}_1;\bar{E}) \cdot \delta V(\boldsymbol{q}_1-\boldsymbol{q}_2)\cdot \tilde{G}_c(\boldsymbol{q}_2,\boldsymbol{p}';\bar{E}) \tag{8}$$

and generalizations with more than one insertion of an interaction potential δV. Alternatively, one can solve the Schrödinger equation (7) with the NNLO potential rather than the LO potential exactly. The two methods are equivalent at NNLO, but differ by higher order terms. This completes the calculation of the resummed cross section.

4. TOP QUARK MASSES

A plot of the threshold cross section displays a large NNLO correction – see the lower panel of Fig. 1 below – that affects both the normalization of the cross section and the position of what is left over from the 1S resonance. It therefore impacts directly on the accuracy with which the top quark mass can be determined from a future measurement. It is worth thinking about the origin of this large correction [9].

We have implicitly assumed that the top quark mass is renormalized on-shell. The NRQCD Lagrangian (4) refers to this choice, but we could have added a small mass term $\delta m_t\psi^\dagger\psi$. The only requirement is that $\delta m_t \sim m_t v^2$ or smaller, so that it is of the same order or smaller than the leading order terms in the NRQCD Lagrangian. This option turns out to be useful in combination with the observation that the large NNLO correction to the cross section peak position is caused by large perturbative corrections to the coordinate space Coulomb potential in the Schrödinger equation. The large corrections arise from loop momentum smaller than $m_t v$ and we can absorb them into the quantity

$$\delta m_t(\mu_f) = -\frac{1}{2}\int\limits_{|\vec{q}|<\mu_f}\frac{d^3\boldsymbol{q}}{(2\pi)^3}\,[\tilde{V}(q)]_{\text{Coulomb}}, \tag{9}$$

where $[\tilde{V}(q)]_{\text{Coulomb}}$ is the Coulomb potential in momentum space. Then define the subtracted potential

$$V(r,\mu_f) = V(r) + 2\delta m_t(\mu_f), \tag{10}$$

which should have smaller perturbative corrections. Since $\delta m_t(\mu_f)$ is r-independent, it is a legitimate mass subtraction, provided we choose the subtraction scale at most of order $m_t v$. (Beyond NNLO the Coulomb potential contains a logarithm as discussed above. To define $\delta m_t(\mu_f)$ completely beyond NNLO, one then has to specify the scale of this logarithm in addition to μ_f.) Hence we rewrite the Schrödinger equation (7) identically in terms of the subtracted potential and $E = \sqrt{s} - 2\,m_{t,\text{PS}}(\mu_f)$, where

$$m_{t,\text{PS}}(\mu_f) \equiv m_t - \delta m_t(\mu_f) \tag{11}$$

defines a new renormalized mass parameter, the potential-subtracted (PS) mass [9].

This reparametrization of the threshold cross section in terms of the PS mass rather than the pole mass should remove the large higher-order corrections to the peak position – and we shall see later that it does. The real benefit, however, is that the improved convergence should occur as well in other processes involving nearly on-shell top quarks. The argument goes as follows: the integrals that relate the top quark pole mass to the $\overline{\text{MS}}$ mass (or bare mass, for that matter) also lead to large higher-order corrections from loop momentum smaller than m_t [16]. One can show by explicit calculation at one loop [9,17], and by diagrammatic arguments in higher orders [9], that the dominant corrections to m_t and the coordinate space Coulomb potential are identical, so that they cancel between m_t and $\delta m_t(\mu_f)$ in (11), when $m_{t,\text{PS}}(\mu_f)$ is related to the $\overline{\text{MS}}$ mass $\bar{m}_t = m_{t,\overline{\text{MS}}}(m_{t,\overline{\text{MS}}})$. Hence any process that is not as sensitive to long-distance corrections as the pole mass and the coordinate space potential separately is expected to have smaller perturbative corrections when expressed in terms of the PS mass instead of the pole mass. (Of course, in processes which involve top quarks only far off mass-shell one can use $\bar{m}_t$ directly.)

Explicitly, we find the following numerical expressions for the series that relate the pole and PS mass, respectively, to the $\overline{\rm MS}$ mass $\bar{m}_t$ ($\bar{m}_t = 165\,\text{GeV}$ and $\alpha_s(\bar{m}_t) = 0.1083$):

$$m_t = \big[165.0 + 7.58 + 1.62 + 0.51 + 0.24\,(\text{est.})\big]\,\text{GeV} \qquad (12)$$

$$m_{t,\rm PS}(20\,\text{GeV}) = \big[165.0 + 6.66 + 1.20 + 0.28 + 0.08\,(\text{est.})\big]\,\text{GeV}. \qquad (13)$$

The 3-loop coefficients are computed using [18]. The 4-loop estimate uses the 'large-β_0' limit [19]. (Note that a NNLO calculation of the threshold cross section determines the PS mass with a parametric accuracy $m_t\alpha_s^4$. To profit from this accuracy for the $\overline{\rm MS}$ mass requires the 4-loop perturbative relation.) The improved convergence is evident and significant on the scale of 0.1 GeV set by the projected statistical uncertainty on the mass measurement.

The arguments above show that it is advantageous to abandon the on-shell mass renormalization scheme even for pair production near threshold. This is perhaps one of the most important conclusions that emerged from the NNLO calculations. In the following we will use the PS scheme with $\mu_f = 20\,\text{GeV}$. Other choices of μ_f are conceivable. Using the definitions (9,11), the PS masses at different μ_f are easily related.

The PS scheme could be called a 'minimal' subtraction scheme, because it subtracts the large infrared correction, but not more. Other, 'non-minimal', mass definitions can be conceived that fulfil the same purpose (see, for instance, the second reference of [4]).

5. RESULT

The top quark cross section at LO, NLO and NNLO (including the summation of logarithms at NLL) is shown in the upper panel of Fig. 1. (To be precise, the NLO curves include the second iterations of the NLO potentials.) For comparison, the result in the on-shell scheme is shown in the lower panel.

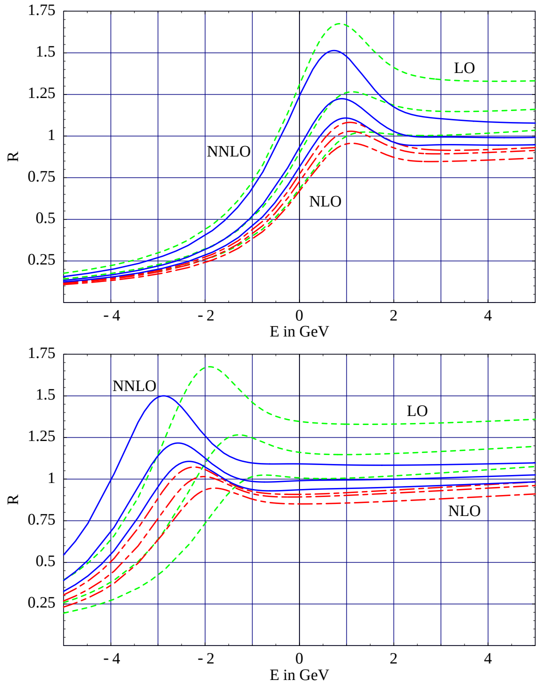

Figure 1. (a) [upper panel]: The normalized $t\bar{t}$ cross section (virtual photon contribution only) in LO (short-dashed), NLO (short-long-dashed) and NNLO (solid) as function of $E = \sqrt{s} - 2m_{t,\rm PS}(20\,\text{GeV})$ (PS scheme, $\mu_f = 20\,\text{GeV}$). Input parameters: $m_{t,\rm PS}(20\,\text{GeV}) = \mu_h = 175\,\text{GeV}$, $\Gamma_t = 1.40\,\text{GeV}$, $\alpha_s(m_Z) = 0.118$. The three curves for each case refer to $\mu = \{15(\text{upper}); 30(\text{central}); 60(\text{lower})\}\,\text{GeV}$. (b) [lower panel]: As in (a), but in the pole mass scheme. Hence $E = \sqrt{s} - 2m_t$. Other parameters as above with $m_{t,\rm PS}(20\,\text{GeV}) \to m_t$.

In the PS scheme, contrary to the on-shell scheme, the peak position varies little, if consecutive orders in the expansion are added. On the other hand, the NNLO correction to the peak height is large, and a significant uncertainty of about $\pm 20\%$ in the normalization remains, larger than at NLO.

The strong enhancement of the peak for the small scale $\mu = 15\,\text{GeV}$ is a consequence of the fact that the perturbative corrections to the residue of the 1S pole become uncontrollable at scales not much smaller than 15 GeV. These large corrections are mainly associated with the logarithms that make the coupling run in the Coulomb potential. This could be interpreted either as an indication that higher order corrections are still important (at such low scales) or that the terms associated with b_0 should be treated exactly, because they are numerically (but not parametrically) large.

If we take (naively) the change in the peak position under scale variations as a measure of the uncertainty of the top mass measurement, we conclude that a determination of the PS mass with an error of about 100 - 150 MeV is possible. Given that the uncertainty in relating the PS mass to the $\overline{\mathrm{MS}}$ mass is of the same order – see (13) – this accuracy seems to be sufficient. It is worth emphasizing that this conclusion does not hold for the top quark pole mass, see (12) and the lower panel of Fig. 1. For a realistic assessment of the error in the mass measurement at a future high-energy lepton collider, the theoretical line shape has still to be folded with initial state radiation, beamstrahlung and beam energy spread effects. Since these effects are well understood, the main question that needs to be addressed is whether the normalization uncertainty leads to a degradation of the mass measurement after these sources of smearing are taken into account. This should be studied in a collider design specific setting.

6. BEYOND NNLO AND OPEN QUESTIONS

The recent developments have put the calculation of the threshold cross section on a more systematic basis. While increasing the parametric accuracy to NNLO, and addressing the issue of mass renormalization in this context for the first time, they have also shown that the theoretical uncertainties are larger than what has commonly been assumed.

The normalization uncertainty is particularly disconcerting. It suggests that yet higher orders in the resummed expansion could be important. One should try to understand whether these large corrections have a physics origin, whether they can be resummed or whether they can be eliminated similar to the large corrections to the peak position in the on-shell scheme.

A complete NNNLO calculation appears to be prohibitive by present standards, primarily because it requires the 3-loop coefficient function of $\psi^\dagger \sigma^i \chi$ and the 3-loop Coulomb potential. But it is already interesting (and possible) to address a well-defined subset of terms. The most obvious subset concerns ultrasoft (retardation) effects, which occur for the first time at NNNLO. They are interesting, because they introduce an explicit sensitivity to the scale $m_t v^2 \sim 2\,\mathrm{GeV}$ and the strong coupling normalized at this small scale.

The sensitivity to the ultrasoft scale exists already in the NLL approximation, in which logarithms sensitive to the ultrasoft scale are summed, with no ultrasoft diagrams to be computed. In addition to the missing inputs to the NRQCD renormalization group, this requires understanding the renormalization group scaling of PNRQCD, which has not been addressed so far. $\alpha_s(m_t v^2)$ then appears as the endpoint of the renormalization group evolution.

Diagrams with one ultrasoft gluon in the Coulomb background represent a true NNNLO correction of order $\alpha_s(m_t v^2) v^2$. The corresponding correction to the 1S toponium energy level and wave function at the origin has already been computed [7]. Near the peak position this constitutes the dominant contribution to the cross section. To assess the numerical significance of this correction, it is necessary to combine it with the NNLL terms that cancel the regulator-dependence of the ultrasoft diagrams.

Accounting for the top quark width correctly represents another challenge. It is an interesting theoretical problem by itself to generalize a non-relativistic effective theory description to the threshold production of unstable particles. Its solution may be useful elsewhere. For the particular case of top quarks, this leads us outside the well-defined framework where only QCD corrections may be discussed. Beyond the leading order implementation of the width currently adopted, one has to consider a more general set of one-loop electroweak corrections. In addition, non-factorizable corrections due to decay products of the top quark interacting with the other top quark or its decay products, and diagrams with single-resonant top quarks, cannot be neglected; the problem has to be formulated in terms of a particular final state such as $WWb\bar{b}$. However, we also expect that these corrections are 'structureless', that is, do not exhibit a pronounced resonance peak. For this reason, we anticipate that they add to the already existing normalization uncertainty, but affect little the top quark mass measurement.

ACKNOWLEDGEMENTS

This summary is based on work done in collaboration with A. Signer and V.A. Smirnov. It is supported in part by the EU Fourth Framework Programme 'Training and Mobility of Researchers', Network 'Quantum Chromodynamics and the Deep Structure of Elementary Particles', contract FMRX-CT98-0194 (DG 12 - MIHT).

REFERENCES

1. V.S. Fadin and V.A. Khoze, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 417 (1987) [JETP Lett. **46**, 525 (1987)]; Yad. Fiz. **48**, 487 (1988) [Sov. J. Nucl. Phys. **48**(2), 309 (1988)]; M.J. Strassler and M.E. Peskin, Phys. Rev. **D43** (1991) 1500.
2. A.H. Hoang and T. Teubner, Phys. Rev. **D58** (1998) 114023; K. Melnikov and A. Yelkhovsky, Nucl. Phys. **B528** (1998) 59; O. Yakovlev, Phys. Lett. **B457** (1999) 170; A.A. Penin and A.A. Pivovarov, [hep-ph/9904278].
3. M. Beneke, A. Signer and V.A. Smirnov, Phys. Lett. **B454** (1999) 137.
4. T. Nagano, A. Ota and Y. Sumino, [hep-ph/9903498]; A.H. Hoang and T. Teubner, [hep-ph/9904468].
5. M. Beneke and V.A. Smirnov, Nucl. Phys. **B522** (1998) 321.
6. M.E. Luke, A.V. Manohar and I.Z. Rothstein, [hep-ph/9910209].
7. B.A. Kniehl and A. Penin, [hep-ph/9907489].
8. M. Peter and Y. Sumino, Phys. Rev. **D57** (1998) 6912.
9. M. Beneke, Phys. Lett. **B434** (1998) 115.
10. W.E. Caswell and G.P. Lepage, Phys. Lett. **167** (1986) 437; B.A. Thacker and G.P. Lepage, Phys. Rev. **D43** (1991) 196; G.P. Lepage *et al.*, Phys. Rev. **D46** (1992) 4052; G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D51** (1995) 1125 [Erratum: *ibid.* **D55**, 5853 (1997)].
11. M. Beneke, A. Signer and V.A. Smirnov, Phys. Rev. Lett. **80** (1998) 2535; A. Czarnecki and K. Melnikov, Phys. Rev. Lett. **80** (1998) 2531.
12. A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. **64** (1998) 428 [hep-ph/9707481].
13. M. Beneke, Talk given at 33rd Rencontres de Moriond: Electroweak Interactions and Unified Theories, Les Arcs, France, 14-21 Mar 1998 [hep-ph/9806429].
14. Y. Schröder, Phys. Lett. **B447** (1999) 321.
15. T. Appelquist, M. Dine and I.J. Muzinich, Phys. Lett. **69B** (1977) 231; Phys. Rev. **D17** (1978) 2074; N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Rev. **D60** (1999) 091502.
16. M. Beneke and V.M. Braun, Nucl. Phys. **B426** (1994) 301; I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. **D50** (1994) 2234.
17. A.H. Hoang, M.C. Smith, T. Stelzer and S. Willenbrock, Phys. Rev. **D59** (1999) 114014; A. Pineda, PhD thesis, Barcelona, 1998.
18. K.G. Chetyrkin and M. Steinhauser, [hep-ph/9907509].
19. M. Beneke and V.M. Braun, Phys. Lett. **B348** (1995) 513; P. Ball, M. Beneke and V.M. Braun, Nucl. Phys. **B452** (1995) 563.