FINGER EXERCISES & BRAIN GYMNASTICS

The art of
M.C. Escher
in the classroom

AKRON
ART
MUSEUM

This collection of activities for classrooms (K-12) will not only examine the mathematics behind Escher's prints, but also provide meaningful mathematical and artistic experiences that will enrich your visit to the Akron Art Museum.

Made popular for his printmaking, M.C. Escher experimented and mastered these techniques including woodblock, mezzotints and lithographs. His prints explore symmetry, tessellations and architecture. Although Escher is best known for his prints of mathematically and geometrically inspired structures and forms, he also created many realistic landscapes in the early part of his career. Escher refers to his early works as "finger exercises" and notes that his later works were much more complex and similar to "brain gymnastics" for him.

His most imaginative works have the ability to bend reality and require the viewer to see the images from new perspectives.

"Only those who attempt the absurd will achieve the impossible. I think it's in my basement...let me go upstairs and check."

-M.C.Escher

ARE YOU REALLY SURE A FLOOR CAN'T ALSO BE A CEILING?

An Introduction to *M.C. Escher: Impossible Realities* Exhibition
February 12, 2011– May 29, 2011

M.C. Escher: Impossible Realities features 130 works by master printmaker Maurits Cornelis Escher, including woodcuts, lithographs, mezzotints, sculptures, and rare preparatory drawings that provide an in-depth view of the artist's creative processes. Featured in the exhibition are seminal and instantly recognizable works such as *Drawing Hands* and *Reptiles*, as well as the extremely rare lithograph stone from the making of *Flat Worms*.

The exhibition comes from the Herakleidon Museum in Athens, Greece, which houses one of the world's largest collections of Escher works. Akron is one of only two North American venues for this extraordinary loan.
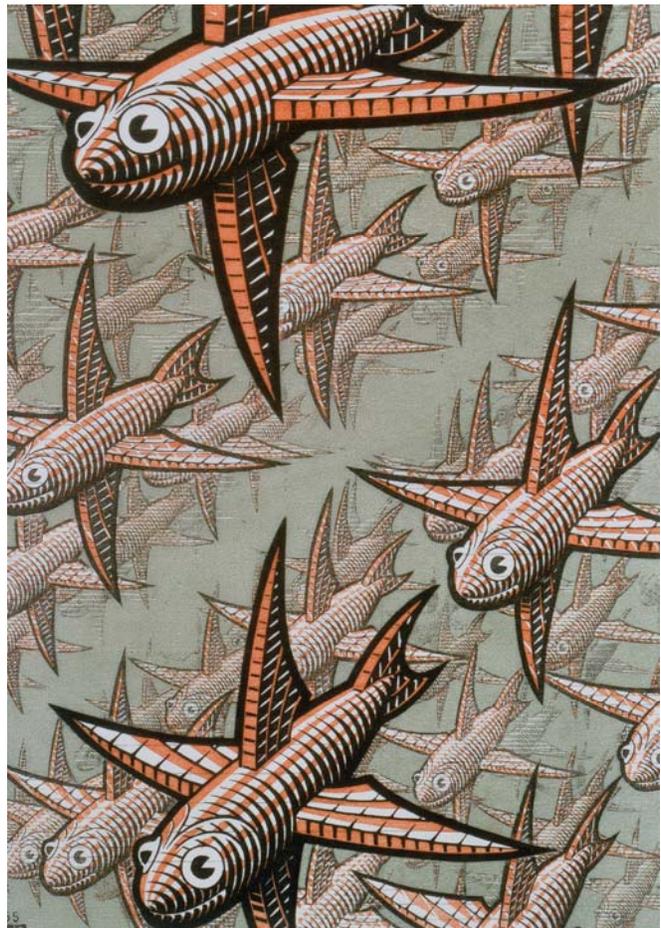
Depth, M.C. Escher, 1955, woodcut and wood engraving

3

WHO IS M.C. ESCHER?
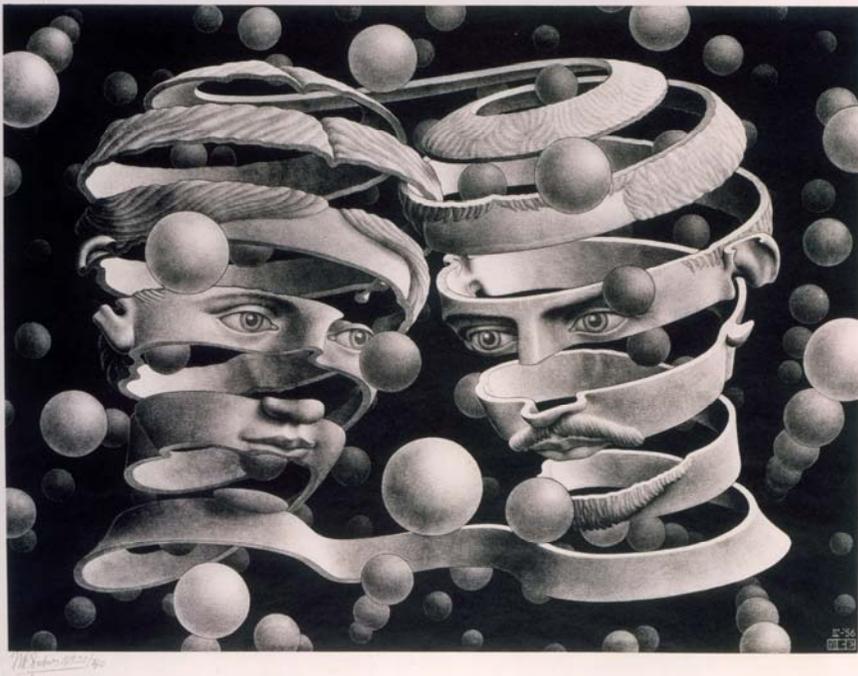
This image, *Bond of Union*, 1956, is a self-portrait of Escher and his wife.

THE BASICS:

Mauritis Cornelius Escher was born in 1898 in Leeuwarden, Holland and died in 1972.

He is most widely known as a graphic artist but was also an illustrator, master printmaker, designer and muralist.

M.C. Escher was interested in drawing and art from the time he was a child. Because of this interest, he went to school for architecture, which involved both drawing and mathematics. However, he decided he liked art much more and studied graphic arts instead.

Escher's career started with book illustrations and landscapes during the 1920s. In the late 1920s and 1930s, he married his wife, Jetta, and traveled throughout Europe. During this time, Escher found inspiration for his artwork in the landscapes of Italy and in the decorative tiles and architecture in Spain.

After settling in Holland with his wife and three sons in 1941, Escher became more and more interested in space, illusion and math. He studied these subjects and created the work he is best known for, such as images of impossible worlds, repeating patterns and reality-bending shapes. These ideas interested him so much, he created artwork about them until 1968.

Even after his death in 1972, Escher's work and ideas continue to excite and inspire people all over the world. His artwork can be seen in museums, books, posters, on television and in movies.

AKRON ART MUSEUM

IMPOSSIBLE TERMS
RELATED TO M.C. ESCHER

Symmetry: in art and design, it refers to when one side balances the other, like drawing a line down the middle of your face and noticing that each side is identical. There are different types of symmetry; including *bilateral symmetry,* which is when two sides surrounding a dividing line (like the face example) are exactly the same. *Radial symmetry* is when the symmetry revolves around a central point like a sea urchin or a tire.
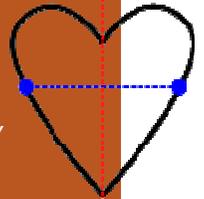
Tessellation: a pattern formed by repeated shapes over a surface without any gaps or overlaps. A lot like soccer balls and a bee's honeycomb.

H.S. M. Coxeter: a mathematician who studied geometry and was inspired by the beauty of mathematics. His discovery "Coxeter Groups" eventually be came known as tessellations. In addition to math, Coxeter loved music and thought of math as a combination of music and symmetry. Escher was actually in contact with Coxeter and he used the Coxeter Group diagrams to help create his drawings.

Crystallography: an experimental science that studies how atoms are arranged in solids.

Mezzotint: A method of engraving a copper or steel plate by scraping and burnishing areas to produce effects of light and shadow.

Lithograph: A printing process in which the image to be printed is drawn on a flat stone surface and treated to hold ink while the negative space is treated to repel ink.

Woodcut: Escher's method of choice, it involves carving and engraving into a block of wood, covering the wood with ink and then printing it.

Infinity/Droste Effect: Notice the "Land O Lakes" butter packaging: The image has a smaller version of itself in a smaller version of itself and so on to infinity. Another way to look at the Droste effect would be with two mirrors. If you have one mirror in front of you and one behind you it will look like it creates a tunnel that goes on infinitely.

Möbius Strips: a surface that has only one side and one edge. Your finger could move over every surface of the strip without crossing over any edge.

Penrose Stairs: made culturally relevant by the recent film, *Inception*, Penrose stairs depict impossible construction; strairs descending at the same time they are ascending!
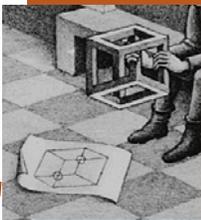
Neckar Cube: simply put, it is a line drawing of a cube. The "impossible cube" illustrated in Escher's *Belvedere* plays with the idea of perception, the line drawing of the cube can be seen facing either direction.

Polygon: 3-D geometric solid that has flat faces or sides and straight edges.

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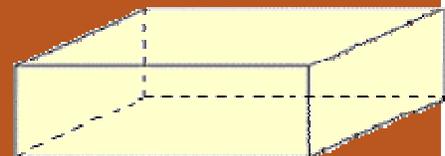
MATH LESSONS

The next few pages will layout multiple lessons that will explore symmetry, geometry, architecture and how to utilize the prints of M.C. Escher in the math classroom. Each lesson includes a hands on portion as well as useful handouts.

"I doubt that 'the public' will ever understand, much less appreciate, how many **gymnastics of the brain**, fascinating to me, have preceded the construction of such a picture."

-M.C.Escher

Math section prepared by Dr. Joanne Caniglia
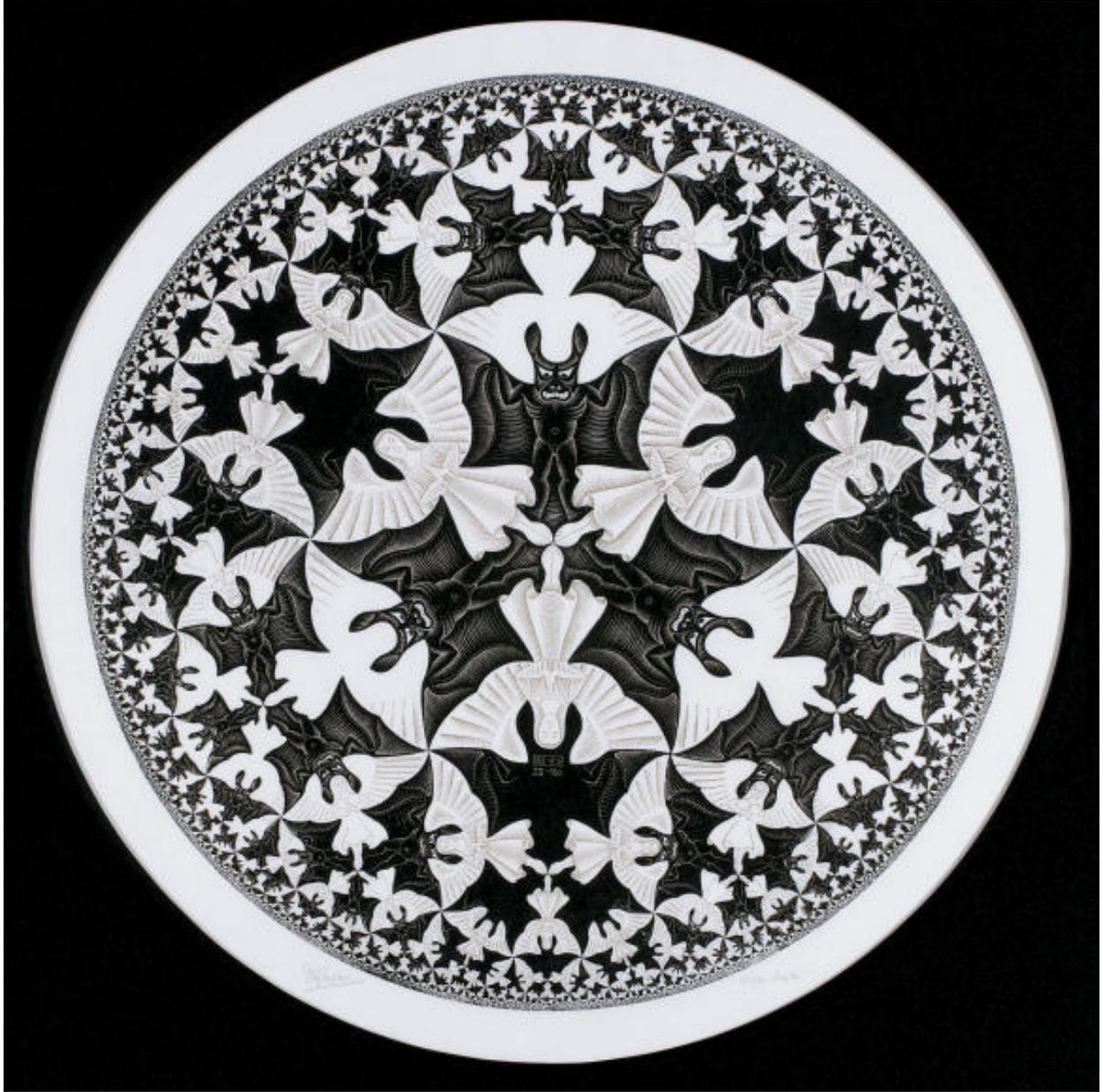
Circle Limit IV: Heaven and Hell, M.C. Escher, 1960, woodcut

Lesson 1 SYMMETRY

Objectives:

Grade 7:
- Identify the line and rotation symmetries of two-dimensional figures to solve problems.
- Perform translations, reflections, rotations, and dilations of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).

Grade 8:
- Draw the results of translations, reflections, rotations and dilations of objects in the coordinate plane, and determine properties that remain fixed; e.g., lengths of sides remain the same under translations.
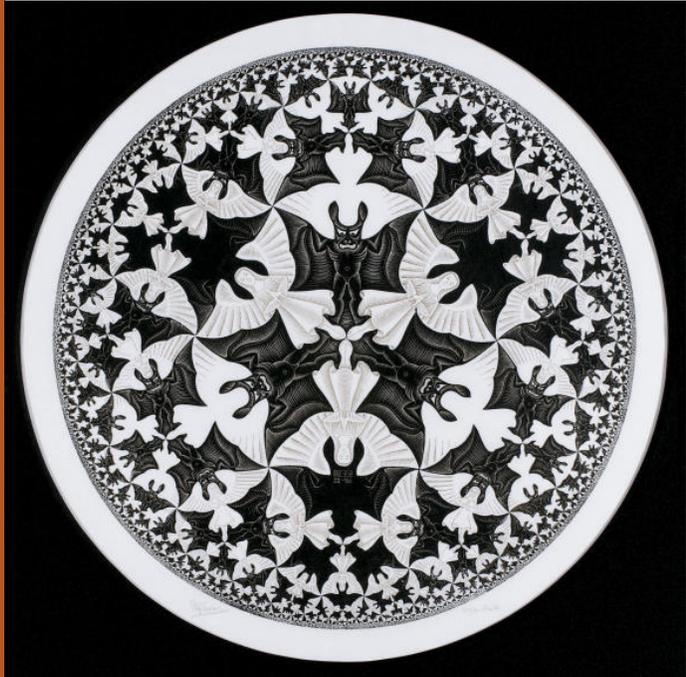
Circle Limit IV: Heaven and Hell, 1960, woodcut

Circle Limit IV, is one of Escher's attempts at constructing infinity within a finite circle. Escher said that what he achieved was not a literal representation of infinity, "but certainly a fragment of it."

The symmetries studied and found in Escher's regular division of the plane are *reflection symmetry*, *rotation symmetry*, *translation symmetry*, and *glide reflection symmetry*. This lesson investigates the basic understandings of symmetry transformation and their relationship to *Circle Limit IV.* The first pages of this lesson define the symmetry found in Escher's *Circle Limit IV.*
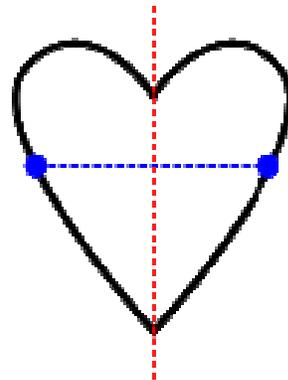
THE MATH BEHIND THE ART

Reflection Symmetry

If points of a figure are equally positioned about a line, the figure has **reflection symmetry**, or **mirror symmetry**. The line is called the **reflection line**, the **mirror line**, or the **axis of symmetry**. The axis of symmetry separates the figure into two parts, one of which is a mirror image of the other part. The simplest case of reflection symmetry is known as **bilateral symmetry**. For example, each of the following figures exhibits bilateral symmetry:
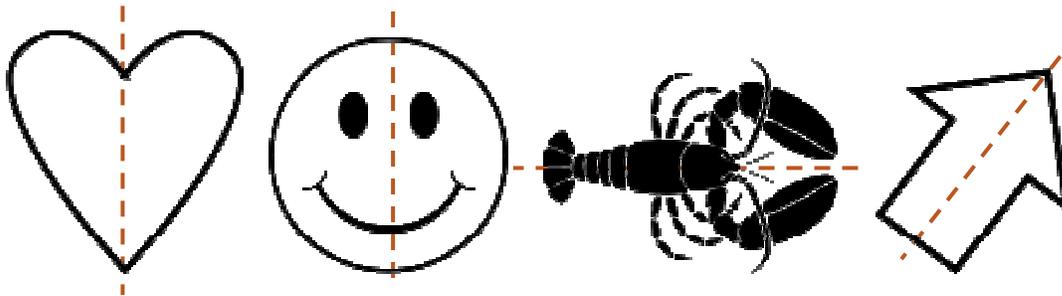
The heart and smiley each have a vertical axis of symmetry, and the lobster has a horizontal axis of symmetry. The arrow has an axis of symmetry at an angle. If you draw the reflection line though any one of these figures, you will notice that for every point on one side of the line there is a corresponding point on the other side of the line. If you connect any two corresponding points with a segment, that segment will be perpendicular to the axis of symmetry and bisected by it (cut into two equal length segments):
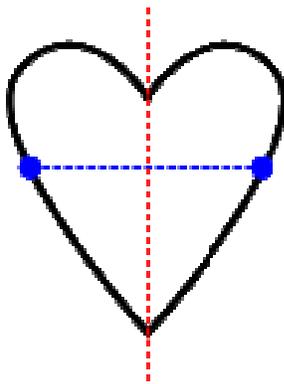
Bilateral symmetry is the most common type of symmetry found in nature, occurring in almost all animals and many plants. Cognitive research has shown that the human mind is specially equipped to detect bilateral symmetry. In fact, humans are especially good at detecting bilateral symmetry when the axis of symmetry is oriented vertically. As you proceed through this course, you will look for symmetry in all sorts of complicated images. Remember that your eyes are hard wired to do this well when the axis is vertical, and so it will be a tremendous help to rotate the images (or your head) as you look for symmetries.

Some objects or images can have more than one axis of reflection symmetry. Here are some examples, with the reflection axes shown as dotted red lines:
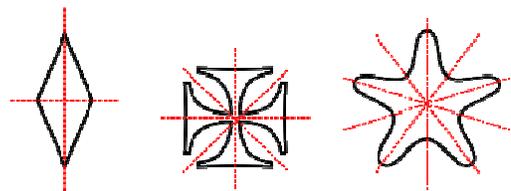
(Pay special attention to the diagonal reflection axes in the cross. These are easy to overlook.)

Lesson 1: Symmetry continued

Rotational Symmetry

If points on a figure are equally positioned about a central point, the object has **rotational symmetry**. A figure with rotational symmetry appears the same after rotating by some amount around the center point. The **angle of rotation** of a symmetric figure is the smallest angle of rotation that preserves the figure. For example, the figure on the left can be turned by 180° (the same way you would turn an hourglass) and will look the same. The center (recycle) figure can be turned by 120°, and the star can be turned by 72°. For the star, where did 72° come from?
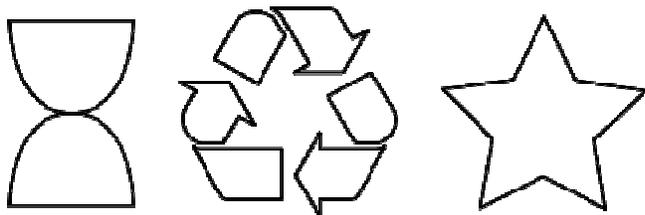
The star has five points. To rotate it until it looks the same, you need to make 1 / 5 of a complete 360° turn. Since, this is a 72° angle rotation.

Translation Symmetry:

To translate an object means to move it without rotating or reflecting it. Every translation has a direction and a distance.

Glide Symmetry:

A *glide reflection* is a symmetry transformation that consists of a translation (movement) followed by a reflection across the translation line. The following examples will illustrate glide symmetry:
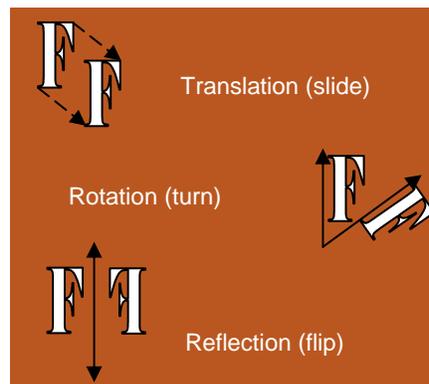
Summary:

A figure, picture, or pattern is said to be symmetric if there is at least one symmetry that leaves the figure unchanged. For example, the letters in ATOYOTA form a symmetric pattern: if you draw a vertical line through the center of the "Y" and then reflect the entire phrase across the line, the left side becomes the right side and vice versa. The picture doesn't change.
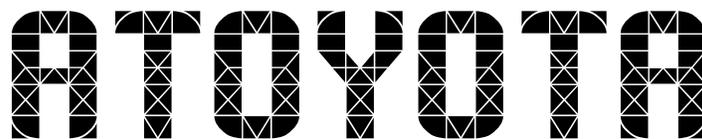
If you draw the figure of a person walking and copy it to make a line of walkers going infinitely in both directions, you have made a symmetric pattern. You can translate the whole group ahead one person, and the procession will look the same. This pattern has an infinite number of symmetries, since you can translate forward by one person, two people, or three people, or backwards by the same numbers, or even by no people.

BRINGING THE MATHEMATICS TO LIFE

Symmetry Scavenger Hunt

Materials

Notebook page to sketch on

Pencil

Moving feet

Escher tessellation reproductions

Procedure

Each group is to find as many examples of symmetry in the Esche Sketches.

For each example that you find, make a quick sketch of it in your notebook and identity the symmetry. An example is found below.

Your whole group must return and present your answers by the end of the project. Example:

| Sketch of Escher Tiling	Type of Symmetry and Name of Print
	Regular Division of the Plane-Red *Reflection and Slide*
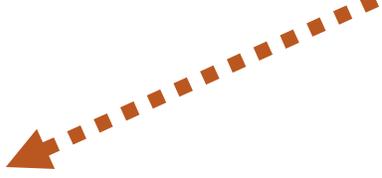
Sketch of Escher Tiling	Type of Symmetry and Name of Print
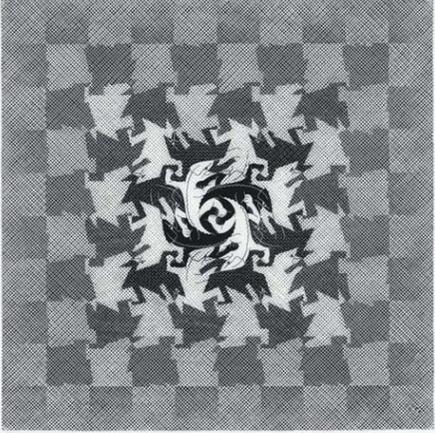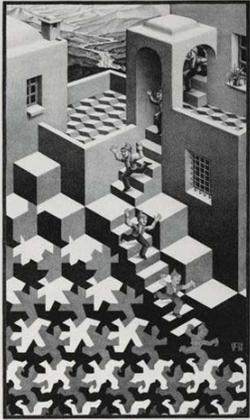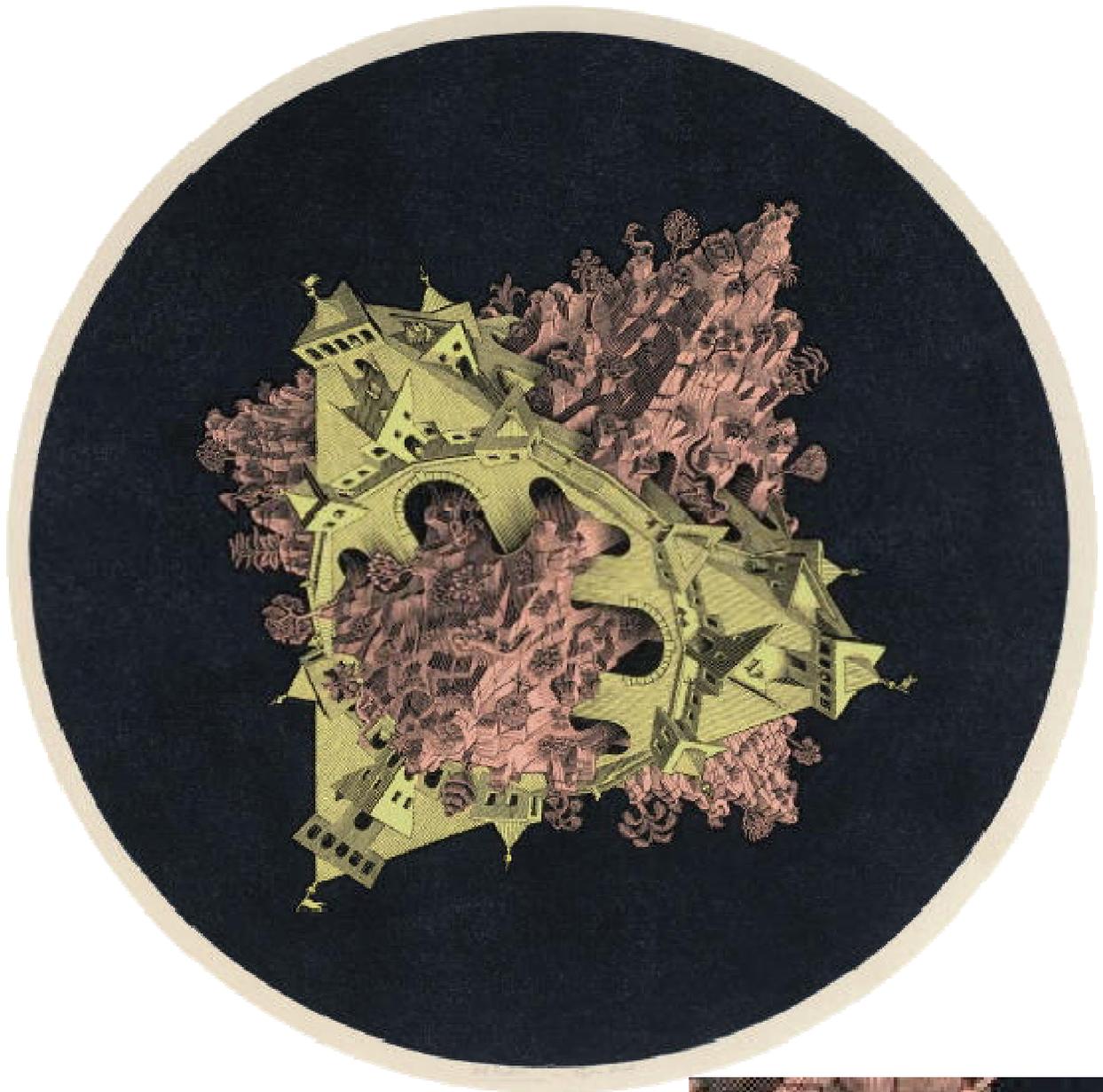
Double Planetoid, M.C. Escher, 1949, wood engraving

Dinosaur!
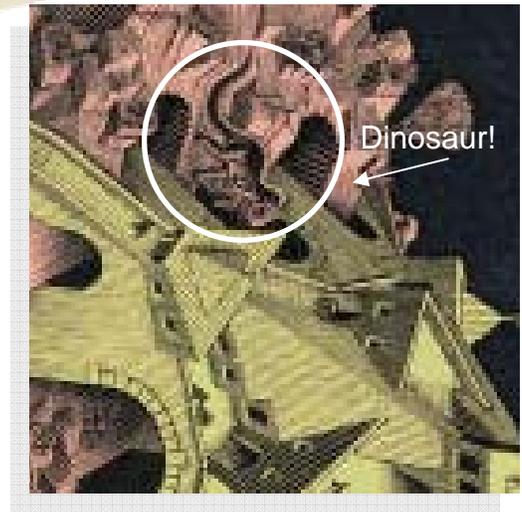
(Detail of *Double Planetoid*)

Lesson 2 GEOMETRY
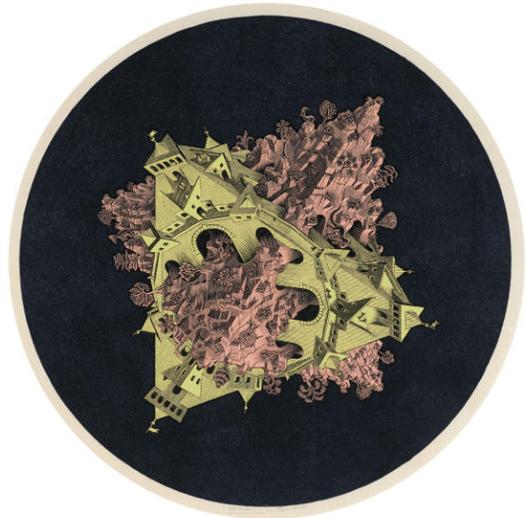
Double Planetoid, 1949, wood engraving

Here you can see two worlds: one inhabited by humans with buildings and the other a prehistoric world populated by dinosaurs. This is an innovative departure from Escher's usual motifs of black versus white and up versus down.

In *Double Planetoid,* you can see an example of a polyhedron.   A polyhedron is a group of polygons attached by their edges. The word *polyhedron* is derived from the Greek *poly* (many) and the Indo-European *hedron* (seat).

Polyhedra can be seen! Crystals are real world examples of polyhedra. The salt you sprinkle on your food is a crystal shape of a cube!
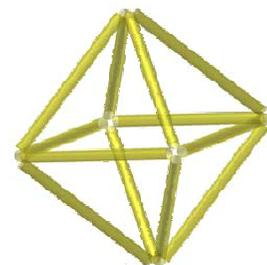
This polyhedron (octahedron) is two pyramids connected at their edges.

BRINGING THE MATHEMATICS TO LIFE

Use simple materials to investigate regular or advanced 3-dimensional shapes. You can make a **Double Planetoid** using straws and paper clips! First make the following shapes.
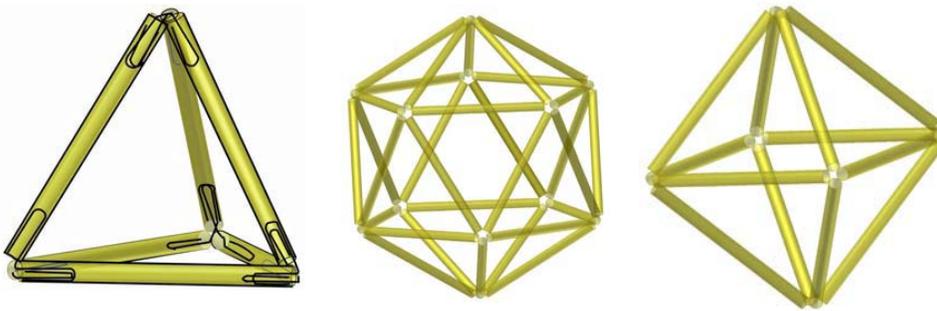
Assembly
Choose which shape to construct. Shapes with triangular faces will form sturdier skeletal shapes; square and pentagonal faces will be less sturdy.
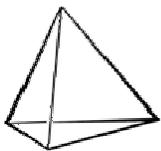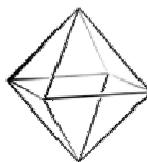
Tetrahedron   Cube   Octahedron   Dodecahedron   Icosahedron
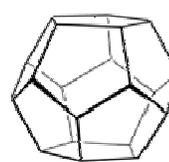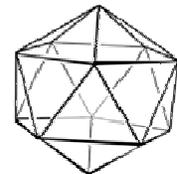
Bend the paperclips so that the 2 loops form a "V" or "L" shape as needed, widen the narrower loop and insert one loop into the end of one straw half, and the other loop into another straw half.

Use the table and images above to construct the selected shape by creating one or more face shapes and then add straws or join shapes at each of the vertices:

Polyhedron	Faces	Shape of Face	Edges	Vertices
Tetrahedron	4	Triangles	6	4
Cube	6	Squares	12	8
Octahedron	8	Triangles	12	6
Dodecahedron	12	Pentagons	30	20
Icosahedron	20	Triangles	30	12

Belvedere, M.C. Escher, 1958, lithograph

Lesson 3 ARCHITECTURE

Objectives:

Grade 4:
- Build a three-dimensional model of an object composed of cubes; e.g. construct a model based on an illustration or actual object.

Grade 5:
- Identify, describe and model intersecting, parallel and perpendicular lines and line segments; e.g. use straws or other materials to model lines.

Here is a man seated at the foot of the building holding an impossible cube. He appears to be constructing it from a diagram of a Neckar cube at his feet with the intersecting lines circled.
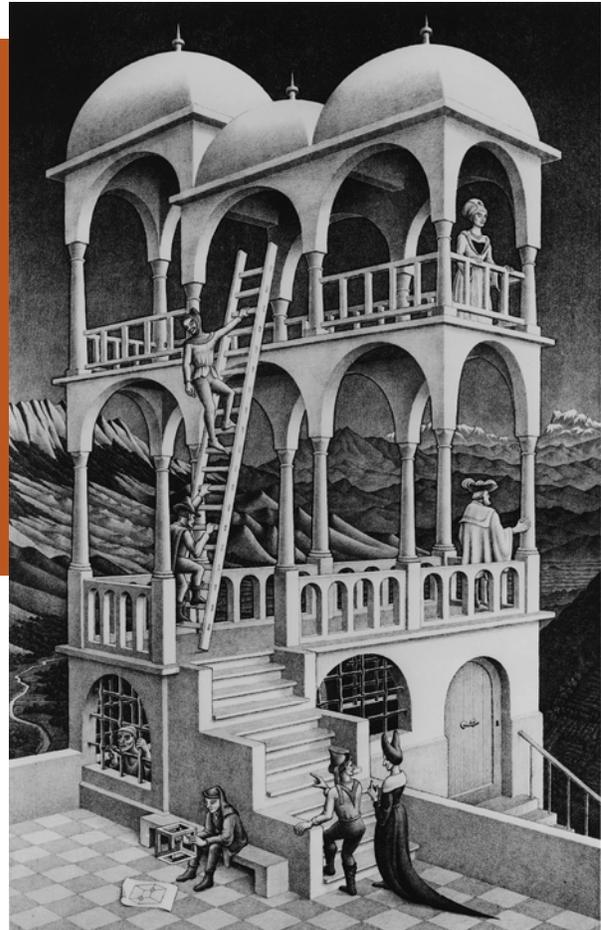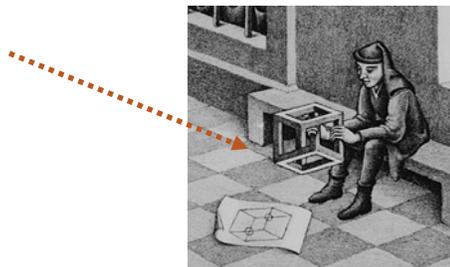
Belvedere, 1958, lithograph

This print shows a plausible-looking building that turns out to be impossible! In this print, Escher uses two-dimensional images to depict objects free of the confines of the three-dimensional world. The image is of a rectangular three-story building. The upper two floors are open at the sides with the top floor and roof supported by pillars. From the viewer's perspective, all the pillars on the middle floor are higher. The viewer also sees by the corners of the top floor that it is at a different angle than the rest of the structure. All these elements make it possible for all the pillars on the middle floor to stand at right angles, yet the pillars at the front support the back side of the top floor while the pillars at the back support the front side. This paradox also allows a ladder to extend from the inside of the middle floor to the outside of the top floor.

BRINGING THE MATHEMATICS TO LIFE

- ## Activity One: Intersecting and Parallel Lines

Materials:

Red and Blue Markers

A reproduction of *Belvedere,* for each student

Directions:

In the print, *Belvedere*, identify as many pairs of parallel and perpendicular lines by tracing over each type of line using a different color marking pens.

- ## Activity Two: Using Isometric Paper to Create Cubes

Materials:

Picture of Necker Cube

Isometric Paper

"MultiLink" Cubes

Directions:

Using the Isometric Tool by the National Council of Teachers of Mathematics :

http://tinyurl.com/yfymp8

Students will try to draw the following figures using both the isometric paper and online tool. Are the figures possible? Try building them with multilink cubes.
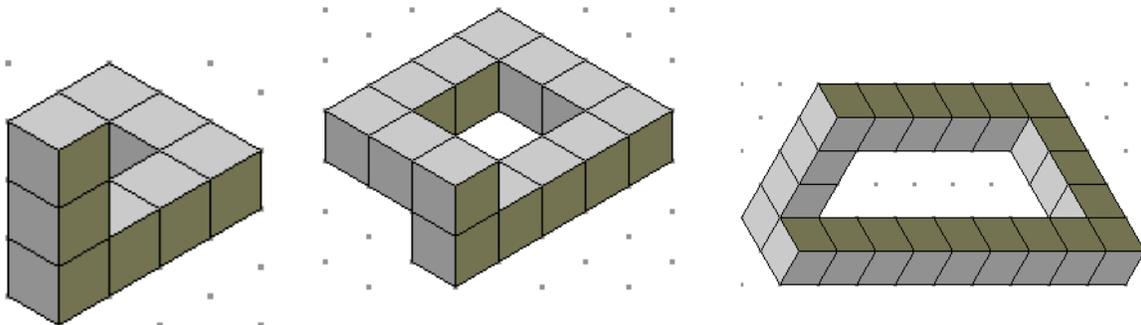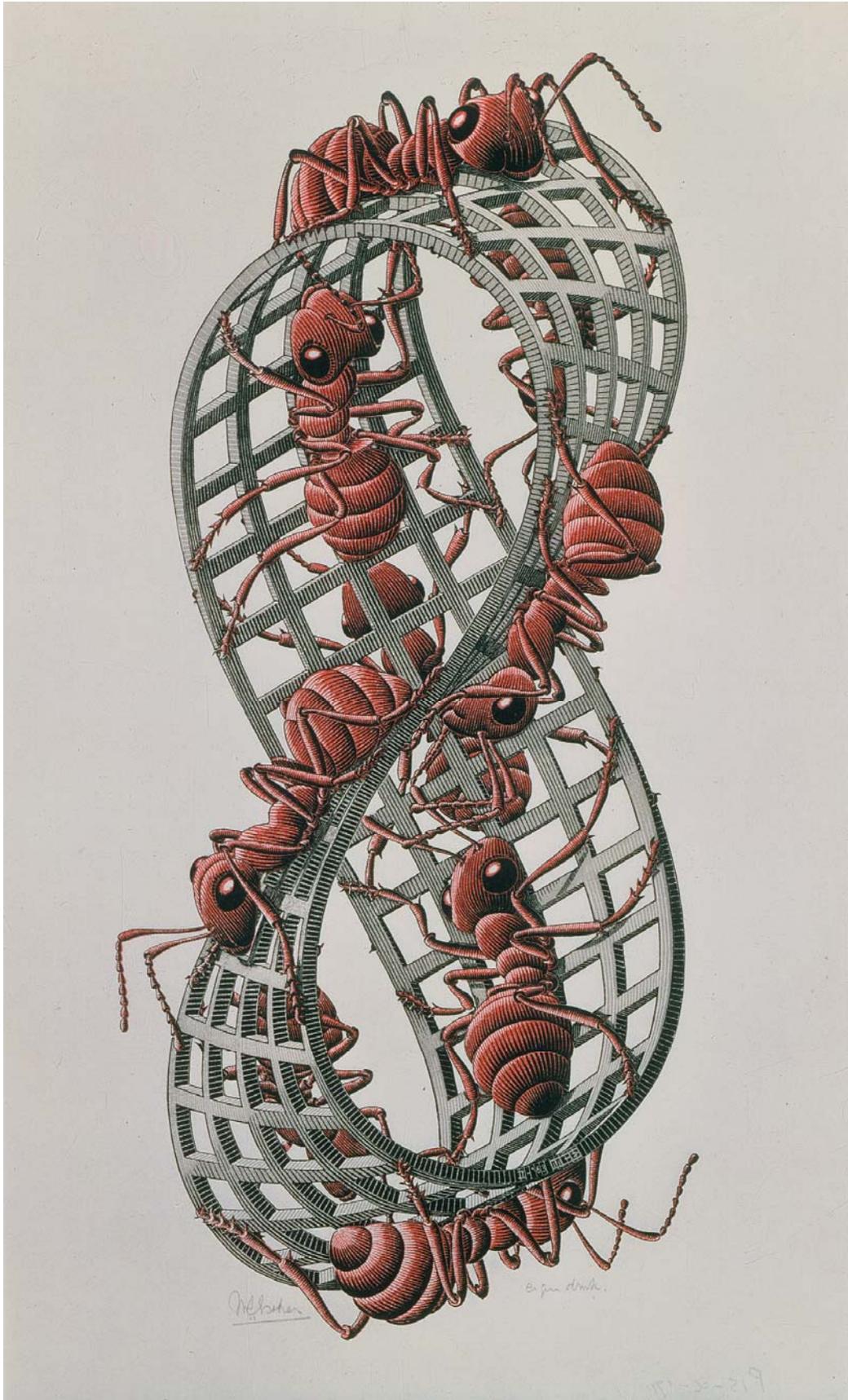
Möbius Strip II (Red Ants), M.C. Escher, 1963, woodcut

Lesson 4 REASONING

Möbius Strip II (Red Ants), M.C. Escher, 1963, woodcut

The Möbius strip was first developed and defined by the German mathematician August Ferdinand Möbius in 1858. In essence, this strip is an easily constructed topological (which refers to the study of geometric space) paradox: **A surface with only one side and one edge!** Here red ants scurry around a Möbius strip. The work refers to Escher's interest in duality.

Imagine a piece of paper. Clearly, this surface possesses two sides and four edges, creating a perfectly standard shape, easily defined in a topological sense. Now, imagine, the same shape, but with three of the edges taken away and with only one side. *Does this seem possible?*

BRINGING THE MATHEMATICS TO LIFE

Making your own Möbius Strip:

For this activity, you will need five strips of paper per student. Each strip should be approximately 3 x 14 inches. Each student will also need about two feet of tape (either transparent tape or masking tape will work), a pair of scissors and a pen or pencil. Use one strip of paper. Put the two ends together, and give one end a half-twist. Tape the paper together this way, being sure to tape across the entire strip.
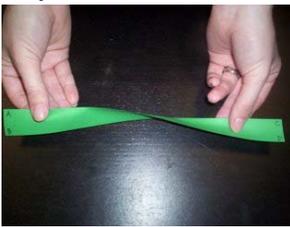

Draw a line down the middle of the strip, continue drawing until you meet up with your starting point.

Take this Möbius strip and cut parallel to the edge about in the middle, right along the line you just drew. What happens? Make another Möbius Strip. If you cut 1/3 of the way from the edge, what will happen? Test your guess. Make another strip, but use a full twist on the end this time, instead of a half-twist. Cut in half. Note what happens. Then, cut again.

Put two of your strips together: one should be a Möbius strip, the other a regular loop. Tape them together at right angles, bisecting each other. What happens if you cut along the middle of both? Try it.

Real-life applications of Möbius Strips:
Ribbons for typewriters or computer printer cartridges are Möbius strips. Also, some belts on cars and farm machinery are put together in this way, in order to provide for more uniform wear and tear on the belts. The universal recycle sign is a Möbius Strip.

SOURCES:
S. Barr, *Experiments in topology,* Dover Publications, NY, 1989
R. Courant and H. Robbins, *What is mathematics*, Oxford University Press, 1996
K. Devlin, *Mathematics: The science of patterns*, Scientific American Library, 1997
C. A. Pickover, http://tinyurl.com/4wgtqot, Thunder's Mouth Press, 2006
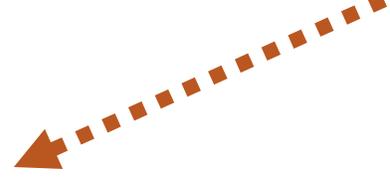
MOBIUS QUESTIONS TO BE ANSWERED

1. Could you have drawn this line without lifting your pencil from the paper? Without turning the paper over?

   _____

   _____

   _____

2. Could an ant walking along your line walk until he met his starting point, without walking over an edge of the paper?

   _____

   _____

   _____

3. How many sides does a Möbius strip have?

   _____

   _____

   _____

5. What do you think will happen if you cut along that line?

   _____

   _____

   _____

ART IDEAS

This section will layout ideas for incorporating installation, printmaking, tessellations, and the artwork of M.C. Escher and Sarah Kabot in the classroom. Each idea for a lesson builds upon the concepts and themes that each artist has investigated. Adaptations to skill and grade level can be made.

"Perhaps all I pursue is astonishment and so I try to awaken only astonishment in my viewers."

-M.C. Escher

24

BUILDING ON IDEAS: MAKING PATTERNS

Reptiles, 1943, lithograph

Among the finest prints Escher produced, *Reptiles*, fascinates us with its clever tessellation drawing from which a lizard crawls to life. You can follow its transformation from the drawn page of tessellations and back again. Tessellations, for Escher, became artistic explorations of math concepts. The open book is Escher's "visual dictionary" in which he systematically recorded every system of interlocking figures.

The word "tessera" in Latin means a small stone cube. They were use to make up "tessellate"- the mosaic pictures that form floors and tilings. Here the term has become more specialized and is used to refer to pictures or tiles, mostly in the form of animals and other life forms, which cover the surface of a plane in a symmetrical way without overlapping or leaving gaps.

Elementary students can explore and understand patterns as well as learn about colors by studying Escher's tessellations. Having students create small stamps out of foam is an easy way for them to create a repetition and pattern. Additionally, stamps will allow students to rotate and translate (math concepts) the shapes and create interlocking patterns. The students can work as a team to create a long tapestry or wall paper full of patterns.

TESSELLATE!

1. How many different kinds of lines can you think of? Think of one line, but not straight line, and draw it from the top right corner to the top left corner of your sponge.

Here are some examples of lines:
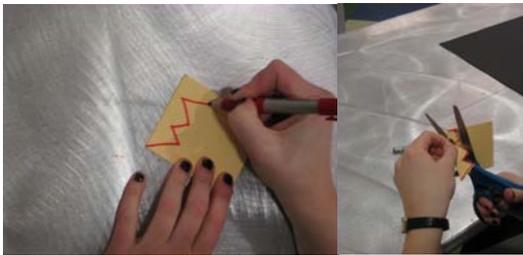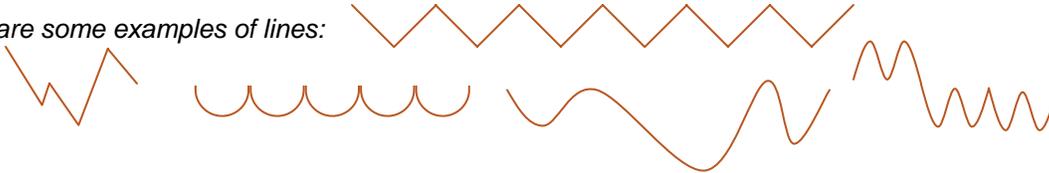
After you have finished drawing your line, cut it out. Now you have two separate shapes that are interlocking, which means they fit together like a puzzle.
Drop your shapes into water, and watch them expand!

2. Dip your sponge shapes into paint. Make sure the whole side of the sponge is covered with paint.

3. Stamp your sponge onto the paper.

4. Use the other shape and dip it into the paint and stamp it onto the paper.

5. Continue your interlocking pattern and alternate colors.
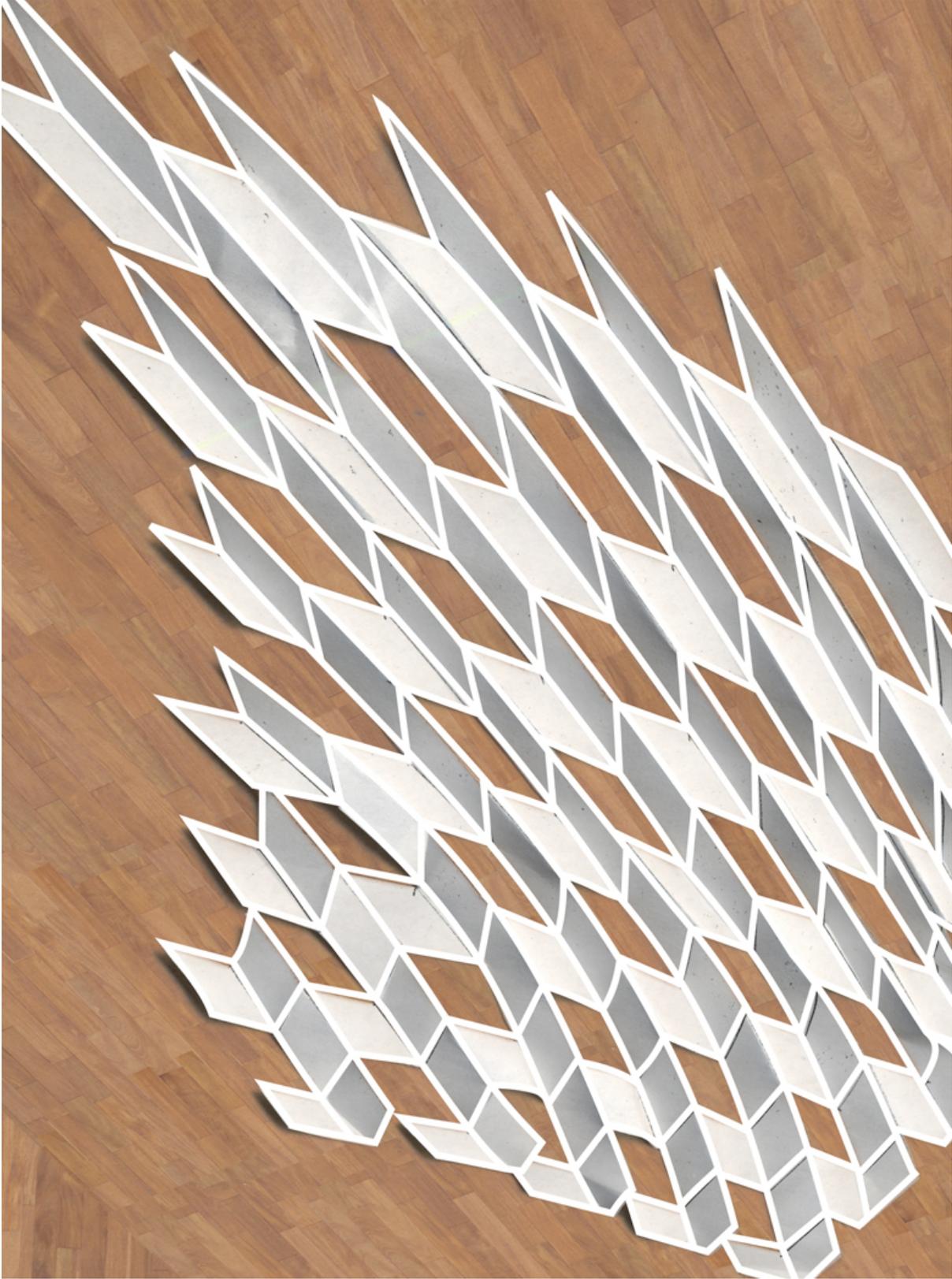
Untitled [preparatory study for *Unfolding Space*], Sarah Kabot, 2010, digital collage

BUILDING ON IDEAS: MAKING SPACE
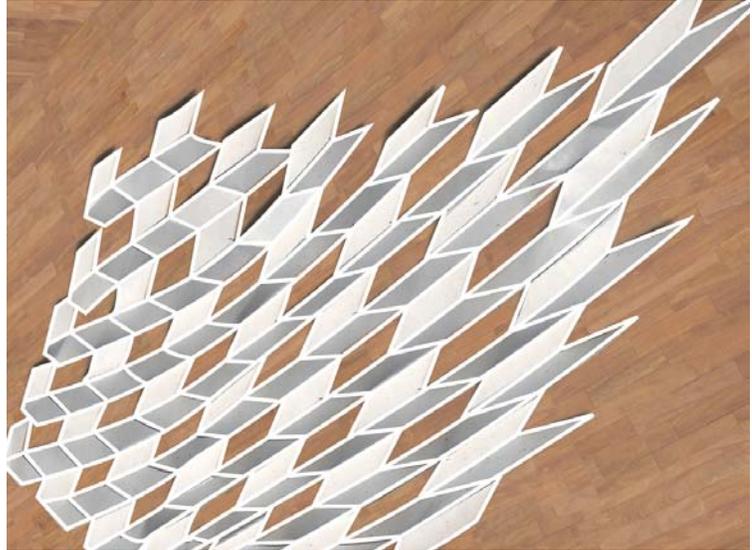
Untitled [preparatory study for *Unfolding Space*], 2010, digital collage

Sarah Kabot creates installations that toy with our perception by altering our physical environment. In *Unfolding Space*, Kabot replicates the lights, floorboards and wall surfaces of the museum's Isroff Gallery and then uses these objects to both deconstruct and reconfigure the space. Much like M.C. Escher did on a two-dimensional surface, Kabot created an interplay between the flat and dimensional, so that the floor will climb up the walls and out into the space of the gallery in the form of steps.

Kabot's interests in perception and illusion intersect with those of M.C. Escher in interesting ways. At the heart of her process is an investigative spirit, a quest to understand the formal structures of familiar objects. Kabot asks us to question what we think we know about the things we see everyday. According to the artist, she is "acting out her imagination of other possibilities for things...what if the ceiling was face-up on the floor?"

How can your students make or change space? Create a lesson that has students creating installations that change the viewer perception on space. Students can take digital photographs of a selected location and then draw on a vellum overlay to depict the changed space. Use the math concepts, such as polyhedron structures to physically create a tent-like construction. To stay on a small scale, a installation could be made with recyclable materials in a diorama.
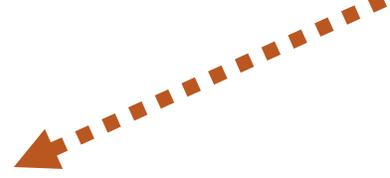
WHO IS SARAH KABOT?

(before) Isroff Gallery

(after) Isroff Gallery,
preparatory drawing for *Unfolding Space*

      Sarah Kabot is an artist who lives and works in Cleveland, Ohio. Kabot creates installations, which are artworks created specifically for a certain space. Installation artists create an environment rather than an artwork that could be placed in any location. Sometimes an installation utilizes more than one of your five senses, like smelling *and* seeing. To start, Kabot does drawings and sketches of the space she will be working in. Kabot's process for creating her installations always involves questions. She wants to understand the way everyday objects such as an exit sign or light fixture are constructed. Kabot asks us to question what we *think* we know about the things we see every day. According to the artist, she is "acting out her imagination of other possibilities for things...what if the ceiling was face-up on the floor?"

Key Words related to Sarah Kabot's work:
Installation
Repetition
Copying
Sculpture
Space

Never Think Before You Act, Flor de Pascua, M.C. Escher, 1921, woodcut

BUILDING ON IDEAS: MAKING PAGES

Never Think Before You Act, Flor de Pascua, 1921, woodcut

This woodcut, among others was featured as decorations for the book *Flor de Pascua (Easter Flower)*, a collection of philosophical writings by a close friend of Escher. For this book, Escher created 15 full-page woodcuts. These prints were created in 1921 when the artist was only 23 years old. The prints illustrate Escher's interest in the dramatic use of black and white and contrast.

Students can create linocut prints that illustrate a theme. While creating the linocuts students will explore figure ground relationships as well as decide on a theme together and then create their own interpretation. Creating prints will allow for multiple works. To take inspiration from Escher, the students can learn about the process of bookmaking and compile each student's print and bind them together. Each student will have a complete thematic book as a finished product.
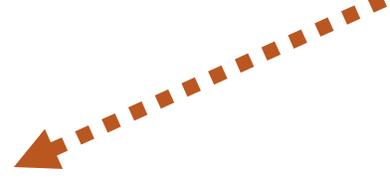
PRINTMAKING FAQ

When preparing to carve a block to be printed…

Brayers (1) are the rollers and they come in various sizes. This tool is used to smooth out the ink and then roll it on to the block. The white plastic spoon is called a **burnisher (2)** and it is used to gently burnish or to polish the back surface of the paper. Burnishing the paper, once the block is pressed on it, will allow the ink to completely transfer onto the paper. **X-acto knives (3)** and **pin tools (4)** can be valuable if you would like to created details in your carving. **Ink (5)** is water soluble or oil-based and comes in many different

Gouge:
The cutting tool used for carving woodblocks or linocuts. Gouges have different blades including one shaped like a C and one like a V; choosing a blade depends on how wide or how narrow you want your carving to be. Always hold the rounded edge of the gouge in the palm of your hand and cut away from you.

When getting your ink ready...

In step 3 when the brayer is ready, the ink is silky and spread evenly across the brayer.