AN APPROACH TO THE DEVELOPMENT OF AN OPTIMAL PROGRAM FOR THE CORRECTION OF DEVIATIONS FROM AN INTERPLANETARY REFERENCE TRAJECTORY

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1956

Submitted to the Faculty of the Graduate School
of the Oklahoma State University of
Agriculture and Applied Science
in partial fulfillment of
the requirements for
the degree of
MASTER OF SCIENCE
August, 1963

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PREFACE

Sometimes a problem exists which, due to convention or other reasons, is usually investigated by classical methods. The classical solution serves, admittedly, as a yardstick with which to compare other methods of analysis. The author feels that this study concerns such a problem. The mathematical simplicity of the "operations analysis" approach to the midcourse guidance optimization problem is impressive as is the large quantity of useful data which it provides. Wider usage of this analytical tool will occur as its versatility is proven through application to many types of problems. Though a distinct novice in the area of operations analysis, the author's objective in this study has been to demonstrate the potential of this method and compare its results to more conventional methods of analysis.

The author is indebted to Professor L. J. Fila for the encouragement and guidance which he gave in this study and in helping to separate the "wheat from the chaff" so to speak. Indebtedness is also felt toward Professor J. R. Norton for the patience and counsel which he gave the author in large amounts. The contribution of Professor W. J. Fabrycky in helping the author understand the nature of operations analysis is gratefully acknowledged. Finally, sincere appreciation is felt toward Mrs. Glenna Banks and Mrs. Dorothy Messenger for the excellent manner in which they brought consistency to the final draft and for their cheerful attitude which served to lighten the author's burden.

May 9, 1963

Stillwater, Oklahoma

James R. Coffee

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LIST OF SYMBOLS

OA	Operations Analysis
A. U.	Astronomical Unit
H	Lawden Cost Function
C	Correction
E	Elliptic Integral
P	Reference Point
P_A	Actual Performance
P_E	Expected Performance
rms	Root Mean Square
R	Ratio of Geometric Progression
TC	Total Cost
W	Word on Computer Data Printout
n	Number of Trips Simulated
v	Velocity of Space Vehicle
v_c	Corrected Velocity
v_{cy}	Component of v_c Parallel to Reference Trajectory
v_{cz}	Component of v_c Perpendicular to Reference Trajectory
v_x	Magnitude of Variable Velocity
x	Random Variable
$t_{\mu, \sigma}$	Random Variable Distributed (μ, σ)

δr	Position Deviation	
δv	Velocity Error	
Δv	Impulsive Velocity Correction	
σ	Standard Deviation of Statistical Distribution	
μ	Mean of Statistical Distribution	
τ	Time Before Arrival	
ϕ	Directional Error Angle	
α	Lawden Program Variable	
β	Observation Reference Angle	
ϵ_β	Angular Measurement Error	
ϵ_c	Clock Error	
ϵ_m	Magnitude Error of Δv	
ϵ_d	Direction Error of Δv	
INJVX	Initial	Distribution Parameters of v_x Error
SIGVX	All Others	
INJDV	Initial	Distribution Parameters of δv Error
SIGDV	All Others	
⊕	Earth Astronomical Symbol	
♂	Mars Astronomical Symbol	
☉	Sun Astronomical Symbol	

Subscripts:

i	Correction Reference Number (i = 1, 2, 3, . . .)
z	Coordinate Perpendicular to Reference Trajectory

y	Coordinate Parallel to Reference Trajectory
L	At Time of Injection into Reference Trajectory
F	At Time of Final Correction

CHAPTER I

INTRODUCTION

The methods of system analysis are useful in the construction of mathematical models for the purpose of determining the response of systems to prescribed inputs. They are also useful when the required response to a given input is specified and it is desired to define the system. In other words, one can utilize these methods for the analytical investigation of existing systems and for the design of new systems.

Webster defines a "system" as "an assemblage of objects united by some form of regular interaction or interdependence". The system function is, therefore, a means to describe this interdependence or interaction with respect to the appropriate units of input and output energy. The system may contain electrical, mechanical, and hydraulic elements. In general, it is possible to define the interaction of these elements in such a manner that a homogeneous set of equations will be obtained. The assumed "lumping" of system parameters serves to simplify the analysis of a complicated system. The ultimate result is often the mathematical model reduced to the lowest terms and capable of yielding the required information to the desired degree of accuracy.

Systems analysis can be thought of as a transformation of a functional requirement into a framework useful to the hardware designer. We would not expect the resultant hardware item to behave precisely as does the mathematical model and for several reasons. First, certain errors or tolerances are inherent in the system components. Secondly, it is not possible to account for all effects of a dynamic environment in the system function. It would be required, however, that certain allowable limits of deviation from the desired value of performance were not exceeded.

It can be seen that from the refined techniques of systems analysis one may often seek to produce a usable item which is not expected to perform in an ideal manner. The system synthesized by these techniques will, on the other hand, behave in an ideal manner. This approach to analysis is useful, however, in that in most cases results are achieved which are compatible with the state-of-the-art in hardware and it is vastly superior to the older "cut and try" methods of design.

With the increasing availability of the high-speed electronic computer, it was to be expected that problems of a broader scope would be subjected to a similar type of analysis. One such class of problems involving a complexity of interdependent subsystems has been studied by the use of a technique called "operations analysis" (OA). One of the basic purposes of OA is to analyze the behavior of such complexes when they are exposed to what can be described as a dynamic environment.

In this context, the term "dynamic environment" infers that certain system variables may occur in a random manner during the analysis.

Whereas OA can be used to approximate the performance of the system complex, the mathematical models available from systems analysis form an integral part of the larger OA model. Each integral subsystem can be thought of as a component of the larger complex and in the same manner the subsystem itself contains a number of components. In the limiting case of simplicity, the OA problem is very like the systems analysis problem. With an increasing complexity of subsystems, however, the point will be reached where it is no longer possible to relate their interdependence to one another by a set of discrete causal relationships. Generally speaking, it is at this point where the OA problem begins.

To some extent, a subtle difference of philosophy also distinguishes OA from systems analysis. The goal of OA is usually directed toward optimizing the utilization of subsystems (resources) by analyzing the effects of their manipulation on the over-all complex. The OA problem is of sufficient scope to justify this sort of analysis. The use of the electronic computer permits a large number of subsystem manipulations to be simulated in a short period of time. The results of this simulation - to a greater or lesser degree depending on the sophistication of the mathematical model - will predict the performance of the system complex.

As compared to systems analysis, OA is not easy to define and often goes under the name of systems analysis. In actuality, relying on Webster again, one can see that systems analysis would more properly be defined as a type of OA since one meaning of "operation" is "an action done as a part of a practical work". The practical work of this paper shall be to determine the optimal corrective program to permit a space vehicle to reach its target. As will be described in Chapter II, the objective is to determine a "policy of operation" which, in this case, is synonymous with "input". Since this is an arbitrarily variable input, however, the distinction should be made. The problem, as solved by OA, will be compared to a mathematical solution. Differences of the results will be discussed as well as the advantages of each.

CHAPTER II

STATEMENT OF THE PROBLEM

The high payload cost for interplanetary vehicles causes serious consideration to be given to methods of minimizing fuel requirements. One would normally expect a space vehicle injected into an interplanetary trajectory to include in its gross weight a quantity of fuel necessary for maneuvers at the destination (i.e., braking, landing, lift-off, etc.), maneuvers upon return to the Earth's vicinity, and maneuvers which may be required en route for the correction of deviations from the desired pre-computed reference trajectories both outward bound and returning.

For practical considerations and for convenience, the analysis of interplanetary missions is often divided into at least three distinct phases which are: 1) Earth escape and/or capture phase; 2) target body capture and/or escape phase; and 3) the midcourse guidance phase. One can thus simplify the analysis by considering a series of two-body problems utilizing Earth-centered and target body-centered coordinate systems respectively for phases 1) and 2) and utilizing a heliocentric coordinate system for phase 3).

2.1 Review of Current Literature

The current literature includes a number of excellent papers dealing with the optimization of space operations in the vicinity of the Earth. (1, 2, 3). In addition, several publications explore the aspects of near-Earth operations with such generality that the methods will be applicable to, say, a Mars approach and landing using the appropriate physical constants. (4, 5, 6). The author wishes to point out that practically all aspects of space navigation and guidance have been and are being handled with such a high degree of mathematical sophistication as to be beyond the scope of this paper. The area of midcourse guidance has not been neglected in this respect. A somewhat simplified approach to the calculation of trajectory corrections was presented by Lawden and Long (7). A further extension of linearized guidance theory by Friedlander and Harry (8) also considered a method of improving guidance logic with each successive correction by applying a scheme of statistical data adjustment and damping coefficients to correction calculations. While these two papers were considering the guidance problem for a ballistic trajectory, Friedlander (9) in a later paper has performed a somewhat similar analysis of a low-thrust trajectory which showed that the midcourse guidance problem was similar for each case.

In retrospect, the foregoing could be misleading if one assumes that the objective of guidance action is to return precisely to the precalculated reference trajectory. In fact, one must establish the desired Keplerian trajectory to serve as the reference for a linearized guidance theory and assume only small deviations from it. With the condition that deviations must be kept small throughout, the criterion for corrective action is that the deviation at arrival must be within predetermined limits. Therefore, for any given corrective action, one is seeking not for a return to the precise reference trajectory but rather for a deviation at arrival to target within certain limits. It should be clear that the solution is, therefore, constrained to the fixed time of arrival of the reference trajectory in problems to which this simplified approach is applied.

2.2 Nature of the Correction Problem

The problem of midcourse guidance is of an iterative nature in that one applies a corrective impulse, waits, observes, applies a corrective impulse and repeats as needed to satisfy the final conditions of miss distance. Though one might assume from earlier discussion that fuel for corrective maneuvers en route makes up only a small share - at least on the outward bound trip - of the total load, the importance of its proper expenditure is great. As demonstrated by Friedlander and Harry (8), using representative instrumentation errors onecould expect an uncorrected deviation at arrival to Mars of approximately 400,000 miles rms[(1)]. It is, therefore, worthwhile that

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[1]The root mean square (rms) deviation is the square root of the arithmetic mean of the squares of the values of deviation obtained from iteration; i.e.

$$\delta r_{rms} = \sqrt{\frac{\sum_i^n \delta r_i^2}{n}} .$$

careful consideration be given to the development of an optimal or least energy cost procedure for midcourse trajectory correction.

It would appear that certain similarities exist between this problem of developing an optimal scheme of midcourse guidance and a representative OA problem such as the development of an optimal inventory policy for an item subject to demand. Fabrycky (10) has investigated this latter problem from the OA approach and has developed a computer algorithm which yields a total item cost surface on which a minimum point can be found. Each point on this cost surface corresponds to certain controllable policy variables which the methods of OA allow one to determine with a given probability. In much the same manner it should be possible to generate a total cost of correction surface with respect to certain guidance policy variables from which a minimum point can be selected. As with the inventory problem, however, it should be kept in mind that there is an associated probability to each total cost point. The context of the term "total cost" is taken as the average total cost per item for inventory policy and the average correction cost per trip for this study.

The initial step should be to describe the nature of the problem and discuss the assumptions which must be made in order to develop the necessary mathematical models. Briefly, the need for a midcourse guidance policy arises because of uncertainties in the inputs to and outputs from the space vehicle's navigation and propulsion systems. It is only necessary to recognize that in the fabrication of physical systems developed by the use of causal theories one is forced to make a concession to reality if he expects to produce items of hardware. This concession to reality recognizes the influence of manufacturing tolerances. Tolerances can be thought of as the upper and lower limits of variation from a desired mean. Systems composed of components subject to tolerances perform according to the summation of these tolerances. The result is that the system performance follows some random pattern of values according to an underlying probability distribution. For convenience, error in the space vehicle guidance and propulsion system will be assumed to occur as a random value drawn from the normal distribution described by the function,

$$1. \qquad f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where x is the distributed variable,

μ is the mean of the distribution, and

σ^2 is the variance of the distribution.

In practice, each of the space vehicle systems is designed and built to perform at a given mean value μ within its prescribed tolerances. Here σ is defined as the standard deviation of the variable x (actual system performance) from μ . The author will use $\pm 3\sigma + \mu$ as the upper and lower limits of the variation of x. System performance falling outside of the 3σ limits will not be considered. This is equivalent to stating that one will ignore those events which have only a probability of .003 of ever occurring since .997 of the area under the normal curve falls within the $\pm 3\sigma$ limits. While these considerations do not materially affect the development of the mathematical model, they are important to the computer program.

2.3 Statement of the Problem

Consider a space vehicle which, as a result of the interaction of the deviations of its component systems from their mean values of performance (hereafter called "error"), is injected into an interplanetary trajectory which deviates somewhat from the desired reference trajectory. For the case of the ballistic trajectory which is discussed in this paper, immediately following injection (attainment of escape velocity) a position fix would be made by star sightings and possibly with earth-based assistance. At a subsequent time another position fix would be made. From this information it will be assumed that the location of the vehicle and its relative velocity can be computed. Based on this position and relative velocity, a corrective thrust application can be calculated to cause the vehicle to satisfy the constraints of fixed time of arrival and miss distance at the target. (7, 8). However, the corrective thrust will be somewhat in error, as will be discovered at a subsequent position fix, and another corrective thrust application will be calculated and applied. Each other succeeding correction will also be in error and must be compensated for until the arrival criteria (time and deviation) are satisfied.

The repetitive nature of the problem is, therefore, evident as well as the need for a correction policy, i.e. "when should a correction be made and what should be the magnitude of the correction?" Any arbitrary values for these variables falling within the capabilities of the space vehicle's systems could be used in theory. The author will attempt to prove, however, that there is a correction policy which will maximize the probability of minimizing the total energy expenditure for corrective action. In the following chapters the simplified mathematical model will be developed and the OA approach to the optimization problem will be utilized.

CHAPTER III

FORMULATION OF THE CORRECTIONAL PROGRAM

3.1 General

An optimal program or schedule for the correction of errors in an interplanetary trajectory has been considered by Lawden (11). He shows that a study of optimization which considers a straight line ballistic trajectory in the absence of gravity yields results which are also valid for the case where one considers a Keplerian trajectory in the presence of gravity. A necessary condition for this similarity is, however, that deviations from the pre-computed reference trajectory be kept small so that second and higher order powers of the deviation or error can be ignored. This is known as the method of perturbations by the use of which a simplified set of linearized guidance equations can be developed. (7, 8).

In this study the validity of the linearized guidance theory will be accepted. The purpose shall be to test the OA method of approach to the problem of optimization rather than development of a non-linear guidance theory. The author is encouraged in this assumption by the volume of current work in the literature which relies on similar assumptions. (7, 8, 9, 11). It is also assumed that the space vehicle is of constant unit mass so that energy expenditure is proportional to the velocity change.

3.2 The Reference Case

The reference trajectory utilized shall be assumed to be a straight line in the absence of gravity with a length equal to that of a given Keplerian trajectory from Earth to Mars. Assumed values of the transfer ellipse parameters are shown below: (8)

TABLE I

Parameter	Assumed Value
Launch Date	December 13, 1964
Trip Time (τ_L), days	192.2
Eccentricity	0.25404
Semimajor Axis, A.U.	1.3059
Length of Trajectory, feet	$7.7943\ (10^{11})$
Average Velocity (v_L), feet/sec	46,936

Calculations of trajectory length and average velocity are shown in Appendix A.

In a study of midcourse correction, Lawden (11) proposes that errors of position determination made from star sightings or position fixes are much smaller than the errors of velocity determination so that the former are considered to be negligible. Utilizing this proposition, it shall be assumed that from a reference location the space vehicle will depart with a velocity subject to error in magnitude and direction following each course correction. At injection, also, this assumption would be made due to the initial injection velocity error. The initial position deviation (δr_L) would be a random value depending on errors occurring during the launch phase.

3.3 Formulation of the Correctional Method

As seen in Fig. 1.a, the geometry of the problem is straightforward. From the initial position (P_{i-1}) the space vehicle will proceed at some velocity (v_i). Figure 1.a shows v_i as a vector of magnitude, v_{xi}, deviating in direction from the reference trajectory by an amount proportional to δv_i (i.e., $\sin \phi_i = \frac{\delta v_i}{v_{xi}}$). The random variable, v_{xi}, is drawn from a distribution of error in the magnitude of $v_{c(i-1)}$ expressed in percent of Δv_{i-1} and δv_i is a measure of the angular deviation of v_i from the reference trajectory. The distributions of v_{xi} and δv_i --which is perpendicular to the reference trajectory--are assumed to be multivariate and normal and will be discussed later in a section devoted to a discussion of the statistical aspects of the OA approach.

The derivation of v_i from the random quantities v_{xi} and δv_i shall be assumed to occur as the result of a position fix. In practice the position fixes would be calculated from astronomical sightings. It is further assumed that the available instrumentation allows the position to be determined prior to the need for correction in every

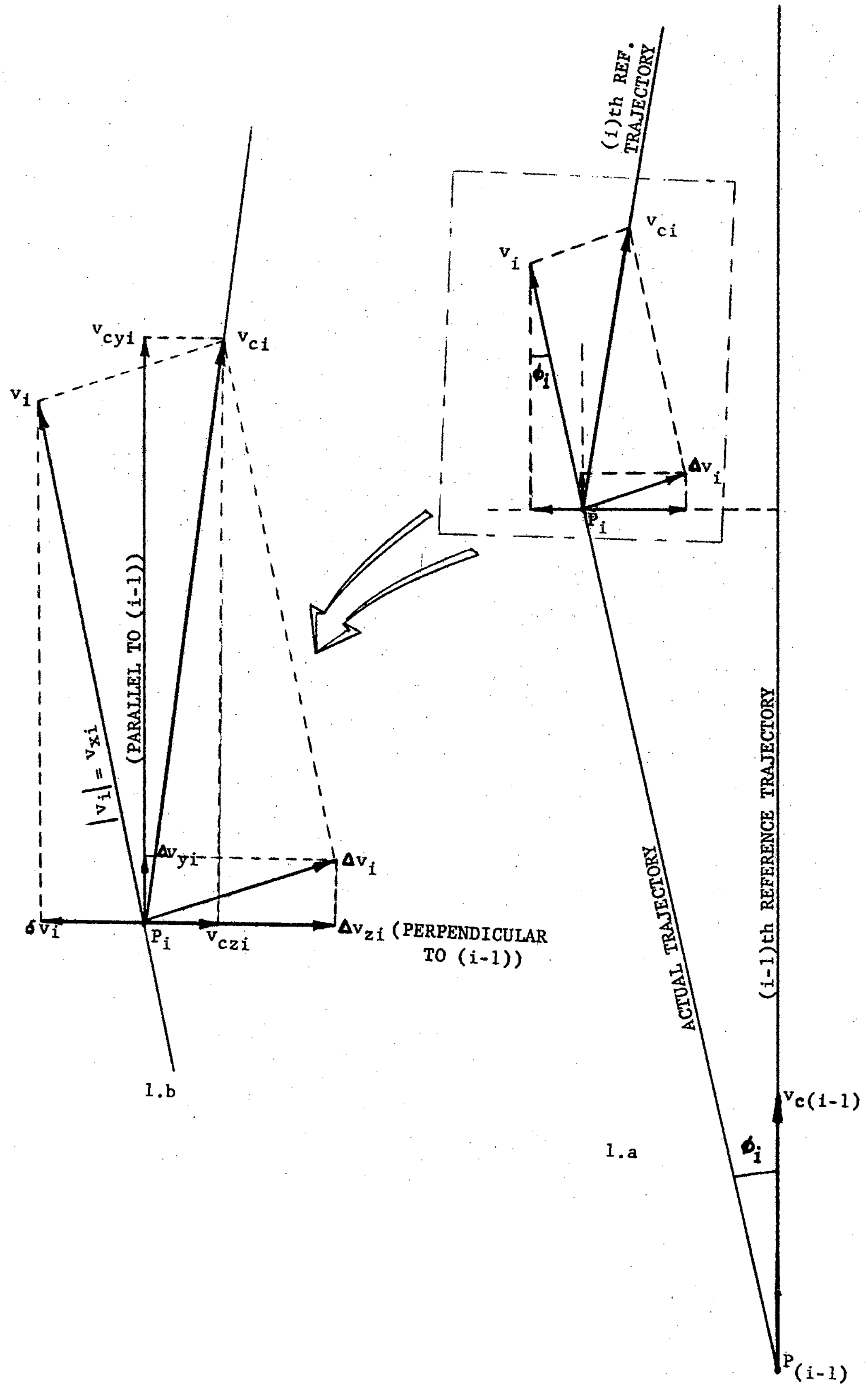

Fig. 1 Geometry of the Correctional Method

case. After v_i is defined, the straight line path actually being followed by the space vehicle from P_{i-1} extended to the point P_i at which point a velocity increment (Δv_i) of the desired magnitude will be required to reduce the miss distance at arrival (δr_F) to zero as a computation criterion. In this manner, Δv_i can be varied parametrically to investigate its effect on total correction cost.

3.4 Computation of the Corrections

Referring again to Fig. 1.b, the computation of the correction velocity v_{ci} can be seen to depend on the desired magnitude of the corrective impulse Δv_i. At each correction the preceding reference velocity $v_{c(i-1)}$ is considered to be along a reference trajectory developed by the (i-1)th corrective action. For convenience, the correction Δv_i is resolved into components perpendicular to and parallel to the (i - 1)th reference trajectory. Since δv_i is known, the necessary component of Δv_i perpendicular to the trajectory is

$$2. \quad \Delta v_{zi} = \delta v_i + \frac{\delta v_i(\tau_{i-1} - \tau_i)}{\tau_i}$$

$$= \delta v_i \left(\frac{\tau_{i-1}}{\tau_i}\right) .$$

The required component of Δv_i parallel to the trajectory is

$$3. \quad \Delta v_{yi} = \left(v_{c(i-1)} - \sqrt{v_{xi}^2 - \delta v_i^2}\right) + \frac{\left(v_{c(i-1)} - \sqrt{v_{xi}^2 - \delta v_i^2}\right)(\tau_{i-1} - \tau_i)}{\tau_i}$$

$$= \left(v_{c(i-1)} - \sqrt{v_{xi}^2 - \delta v_i^2}\right)\left(\frac{\tau_{i-1}}{\tau_i}\right) .$$

It is thus seen that the component Δv_{zi} is determined by and must compensate for two factors. First, Δv_{zi} must nullify the deviation velocity δv_i. Secondly, Δv_{zi} must be of a sufficiently greater magnitude than δv_i but in the opposite direction so that the displacement, $\delta v_i(\tau_{i-1} - \tau_i)$, is also compensated for by the time of arrival at target. By these arguments, also, the component Δv_{yi} must account for the discrepancy of velocity parallel to the reference trajectory and allow the error in displacement due to this discrepancy to be compensated for by the time of arrival.

A little thought about this method of correction will assure one that it is not restricted to a two-dimensional case. There is no restriction on the deviated trajectories causing them to be coplanar with the original reference trajectory or with each other. Neither will the corrected reference trajectories be necessarily coplanar with each other but only with the original reference trajectory. However, each individual correction can be thought of as occurring in its particular two-dimensional coordinate system in the plane established by the corrected reference trajectory from the (i - 1)th correction and the observed velocity of the space vehicle immediately before the (i)th correction.

In Equations 2 and 3, all terms are known except the components of Δv_i and the value of τ_i. Time is of the essence in this type of problem since the energy required to correct a given deviation is inversely proportional to the time remaining to arrival at the target.

The value of Δv_i, which will be varied parametrically, is seen from Fig. 1 to be

$$4. \quad \Delta v_i = (\Delta v_{zi}^2 + \Delta v_{yi}^2)^{\frac{1}{2}} .$$

Therefore,

$$5. \quad \Delta v_i^2 = \delta v_i^2 \left(\frac{\tau_{i-1}}{\tau_i}\right)^2 + \left(v_{c(i-1)} - \sqrt{v_{xi}^2 - \delta v_i^2}\right)^2 \left(\frac{\tau_{i-1}}{\tau_i}\right)^2$$

$$= \left(\frac{\tau_{i-1}}{\tau_i}\right)^2 V,$$

where $$V = \delta v_i^2 + \left(v_{c(i-1)} - \sqrt{v_{xi}^2 - \delta v_i^2}\right)^2$$

and

$$6. \quad \tau_i = \frac{\tau_{i-1}}{\Delta v_i} V^{\frac{1}{2}} .$$

The solution for τ_i determines when the (i)th correction will be made. In this study, errors in time measurement will be taken as small and contributing to the deviation of the multivariate distribution of δv_i. Once τ_i is calculated, the components of Δv_i are easily determined and the new reference velocity v_{ci} can be computed by considering components perpendicular to and parallel to the (i-1)th reference trajectory as follows:

$$7. \quad v_{cyi} = \sqrt{v_{xi}^2 - \delta v_i^2} + \Delta v_{yi}$$

$$8. \quad v_{czi} = \Delta v_{zi} - \delta v_i$$

$$9. \quad v_{ci} = (v_{cyi}^2 + v_{czi}^2)^{\frac{1}{2}} .$$

The method of calculating the (i)th correction can be readily adapted to the digital computer. Each correction is related to the conditions of the previous reference trajectory and the existing error. The first correction (i = 1) is related to the initial deviated reference trajectory. As previously stated, there is random error (δr) in the initial position of the space vehicle at injection. However, since δr_L is very small with relation to the length of the trajectory, no adjustment will be made to the original reference trajectory in making the first correction. The calculation of τ_1, Δv_1, and v_{c1} would be done just as for the (i)th correction and the subscript (i-1) would be replaced by "L". Parameters for the assumed original conditions are shown in Table I.

3.5 General Considerations

The computations described by Equations 2 through 8 make it possible to determine when a correction is required and how the correction will affect the total velocity of the space vehicle. It is clear that the results of this computational method will not be continuous at τ_i equal to zero since by Equation 5 the required correction would be infinite. As τ_i grows smaller, the interval between τ_{i-1} and τ_i will become very short. To account for this, the correctional program will be terminated when $\tau_i <$ one day and the last correction calculated for τ_i = one day. The period of one day is, of course, arbitrary and would actually depend on the deviation observed. However, it can be assumed that any deviation occurring after τ_i = one day is negligible. Actually the last correction could be made at some time less than a day if the requirement for accuracy at arrival warranted. The work of Lawden (11) was considered in establishing this arbitrary value of τ_F. The author of this paper will later make some comparison of his results with those of Lawden and is therefore seeking to maintain a justifiable basis for the comparison of results. As Lawden points out, the final correction should not be confused with the impulse at $\tau = 0$ to transfer from the hyperbolic Mars approach trajectory to a Mars capture orbit. It is, rather, the final correction made to assure the proper position at $\tau = 0$ for initiation of the transfer.

3.6 Applications of the Method

Two types of correctional programs will be calculated by the methods of this chapter. The first to be considered and simulated will be a purely theoretical one wherein no time constraint is placed on the initial correction or any subsequent corrections except for the final one at τ_i = one day. All corrections made in the theoretical program will be of the same magnitude with the exception of the final one. The second type of program which will be simulated is a so-called practical program which does not permit a correction to be made prior to injection. That is, $\tau_1 \leq \tau_L$. In addition, the practical program requires that the last correction be made at τ = one day as discussed previously.

Both the theoretical program and the practical program can be optimized for the constraints imposed on them. By investigating the results of each type of program it should be possible to draw some conclusions which will help to clarify and describe more completely the midcourse correction problem.

In determining the optimal correction policy under the practical program, Δv_i will be varied as a parameter between limiting values. During a particular trip the value of Δv_i will be held constant, however, except for the initial and final corrections. The amount of correction shall be calculated as

$$10. \quad \Delta v_{L,F} = \frac{\tau_{i-1}}{\tau_{L,F}} \quad V^{\frac{1}{2}} \quad ,$$

which is equivalent to Equation 5, except that Δv is the dependent variable. This is a necessary concession to practicality as will be seen later. When small values of Δv_i are used with Equation 5, a τ_i will occur which is greater than τ_L. That is, the equation yields a result indicating the need for a correction prior to injection. In this case, since small values of Δv_i will be of extreme interest, the exact value of the needed correction will be calculated at the instant of injection and this correction will be larger than Δv_i. For larger values of Δv_i when $\tau_i \leq \tau_L$ the first correction will not be made until the full impulse Δv_i is required.

In computing the total cost of correction using Lawden's (11) program, Equation 10 is utilized for calculating each individual correction. The optimal program of Lawden's fixes the times at which corrections will be made by minimizing the function

$$11. \quad H = \sum_{i=1}^{n} \Delta\tilde{v}_i$$

$$= \sum_{i=1}^{n} \frac{\tau_{i-1}}{\tau_i} V^{\frac{1}{2}},$$

where $\Delta\tilde{v}_i$ is always taken as the dependent variable and H is the total cost of correction. The method of this paper will minimize the function

$$12. \quad TC \text{ (Total Cost)} = \sum_{i=1}^{n} \Delta v_i + C_L + C_F ,$$

where Δv_i is the independent variable, C_L is the initial correction, and C_F is the final correction. In this case n is the number of full corrections made of magnitude Δv_i; C_L can be either zero or greater than Δv_i; and C_F is always less than Δv_i.

As stated previously, Lawden has minimized the function for total correction cost by the techniques of the calculus of variations and arrives at a set of values for $\tau_i (i = 1, 2, 3.., 6)$ at which times corrections must be made. The values of τ_i form a geometric progression with the ratio

$$13. \quad \frac{1}{R} = \alpha^{1/n-1} \text{, where}$$

$$14. \quad \alpha = \frac{\tau_L}{\tau_F} .$$

In this reference case, the value of τ_L is 192.2 days and τ_F is one day. The calculations for the parameters **R** and α are shown in Appendix B as well as the computation of the τ_i .

CHAPTER IV

THE NATURE OF ERRORS IN THE CORRECTIONAL PROGRAM

4.1 General

The methods of statistical probability theory are often used to investigate the performance of physical systems. In the real world one expects a system to perform at some approximate level within an acceptable range of values of repeatability which can be defined by the designer or manufacturer and which is a function of manufacturing tolerances or state-of-the-art. When a complex assembly of subsystems is analyzed, the performance of each subsystem for a given event can be drawn at random from a universe or population of performance values for that particular subsystem. The performance values will be distributed according to some frequency function, or at least approximately so. The underlying assumption made for the purpose of analysis is that the frequency function is definable. As stated earlier, in this paper the "normal" frequency function is assumed, for convenience, to define the distribution of actual performance.

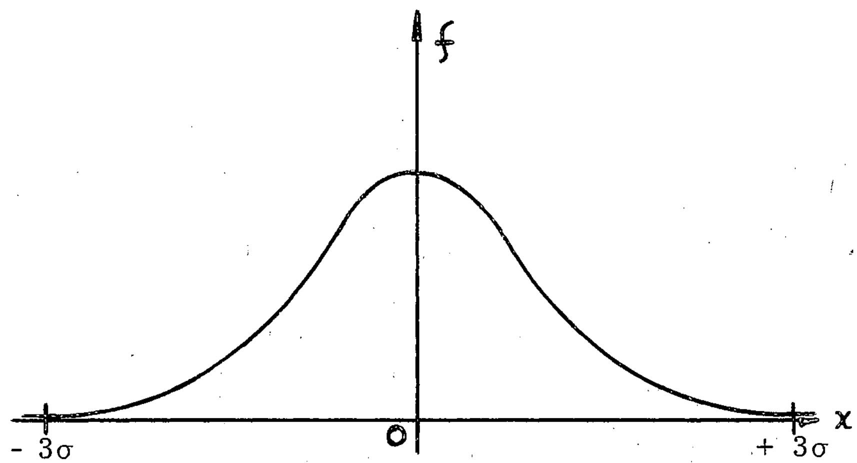

Fig. 2. A Normal Curve

4.2 Properties of the Normal Distribution

The normal frequency curve is asymptotic with the axis of the distributed variable as shown in Fig. 2. In theory, it is assumed that the area under this curve from $-\infty$ to $+\infty$ is equal to unity. The vertical scale of the normal curve is the frequency for the associated value of the variable of interest. Once this distribution is defined, the methods of statistical probability theory can be applied to simplify its use for analysis. Two parameters of special interest are the mean value (μ) and the standard deviation (σ). In the case of physical system performance, one would take μ as the value to be expected and σ to be a measure of the range or variation of the possible values which could occur due to tolerances or error.

In practical applications, however, one does not usually consider all possible values of the variable from $-\infty$ to $+\infty$ due to computational difficulties but often uses the values falling between the $\pm 3\sigma$ limits. Using these limits means that one is considering the range of values for the variable which includes 99.7 percent of the area under the normal curve. Since this area is used as a measure of probability, he is saying, in effect, that there is only a 0.003 probability that a value of the normally distributed variable will occur beyond these limits.

In this study it will be assumed that the $\pm 3\sigma$ limits are defined by the system performance tolerances and that μ is the midpoint or expected value. Where the author deals with error distribution he will take μ to be zero error. In the analysis, then, the performance of the system would be derived by applying a random value of error, drawn from the normal distribution, to the expected value of system performance as follows:

15. $$P_A = P_E (1 + \epsilon_x),$$

where

P_A is actual performance,

P_E is expected performance, and

ϵ_x is the random value of error and can be positive or negative.

4.3 Midcourse Guidance Errors

This preliminary discussion has described a form of Monte Carlo simulation of system performance. In utilizing the OA approach to the problem of this paper, guidance errors will be simulated by means of the Monte Carlo method. Were it not for errors, of course, no corrective program would be necessary. They do exist, however, as previously discussed and they must be observed and corrected. Three types of errors concern one in the midcourse guidance problem and they are: (a) Propulsion errors; (b) Observation errors; and (c) Clock errors. Propulsion errors result in an improper velocity magnitude and direction. Observation errors affect the estimates of the space vehicle's velocity and position. Clock errors contribute to each of the former and could cause computational inaccuracies beyond the midcourse guidance problem.

The previously mentioned quantities of v_{xi} and δv_i can be seen, therefore, to result from multivariate distributions of error in (a), (b), and (c) above. From the literature are taken commonly used values for these errors as shown in Table II. (8,12).

TABLE II

CHARACTERISTIC GUIDANCE ERRORS

Parameter			σ, rms
ϵ_B	=	Observed Angle Error, sec arc	10
ϵ_C	=	Clock Error, %	0.001
ϵ_M	=	Δv Magnitude Error, %	0.1
ϵ_D	=	Δv Direction Error, sec arc	20

Some useful conclusions can be drawn from the consequences of these errors by careful consideration of their relation to the midcourse guidance problem. It is obvious that angular error in observations made of bodies at a distance on the order of magnitude of several A. U.[1] can cause a considerable miscalculation of position. This same error will cause an improper velocity estimate to be made. In general, the small velocity error is of more concern than the small position error because of a limited corrective capability and the desire to optimize the utilization of corrective energy. In addition, the position errors, though seemingly large, are small in comparison to the interplanetary distances involved and as the target is approached these errors decrease due to the smaller observational distance. Clock error would be summed with angular error and would be proportional to the period of time between observations.

The error of Δv is a function of the vehicle's propulsion and orientation systems. These could also be summed with the other errors to yield a complex four-dimensional distribution of velocity error. It is not within the scope of this paper, however, to pursue the error analysis of such correlated observations. Excellent work in this area has been done by others. (8, 13). It is necessary, however, to

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[1]Astronomical unit: The magnitude of semi-major axis of the Earth's elliptical orbit.

relate these errors in some logical way to the problem at hand.

The full utility of the OA approach to determination of an optimal correction program cannot be realized if one is restricted by existing system capabilities. In example, the estimated velocity from observations made a few hours apart could easily be in error by an amount far greater than the maximum possible Δv. The author, therefore, feels justified in this preliminary investigation to relate error in velocity to the preceding impulsive velocity correction. That is, the error to be corrected at τ_i will be taken as proportional to the correction (Δv_{i-1}) at τ_{i-1} where the error components of v_{xi} and δv_i are drawn from distributions which will be defined. The assumption is that the net result of all errors would be in proportion to the size of the correction attempted because the result of these inaccuracies is that one will calculate and apply the correction erroneously.

This simplifying assumption will not seriously restrict the results of this study. In general, the investigation of a problem by OA methods starts with a simplified mathematical model. The model is frequently refined as the complicated interrelationship of the individual systems to one another is studied by simulation. Some areas will be found where minute detail is required while in other subsystems it may be only necessary to simulate a Gaussian output. In the problem of this paper only the simplified model is developed. Virtually every included subsystem is a suitable subject for an individual investigation.

The errors shown in Table II for Δv are directly convertible to a value of resultant velocity. For instance, the random variable v_{xi} in the absence of observational or clock error would be:

16. $$v'_{xi} \cong v_{c(i-1)} + \Delta v \epsilon_M .$$

The variable δv_i, under the same conditions, since the angle is very small would be:

17. $$\delta v'_i \cong \Delta v \sin \epsilon_D .$$

It should be kept in mind that ϵ_M and ϵ_D are random variables drawn from distributions with $\mu = 0$ and values of σ as shown in Table II. Utilizing the 3 σ limits as discussed previously, due only to errors in the application of Δv the resultant velocity would have a magnitude error between $\pm$ 0.003 Δv_i and direction error of $\pm$ 0.0003Δv_i. On the straight line reference trajectory of this problem the miss at arrival due to these small errors alone could exceed 10^4 miles even assuming the target to be stationary.

For this study the author will arbitrarily double the error due to application of Δv_i and assume all errors to be included in the resulting distributions of ϵ'_M and ϵ'_D as shown in Table III.

TABLE III

ASSUMED TOTAL GUIDANCE ERROR

Error	σ
ϵ'_M, % (Δv)	0.2
ϵ'_D, % (Δv)	0.02

These values appear as constants in the computer program of Chapter V with ϵ'_M = SIGVX and ϵ'_D = SIGDV.

The first impulsive velocity change which is subject to error, however, will be assumed to be at injection where the velocity is increased from that of earth orbit to the reference velocity shown in Table I. Since the vehicle would be very near Earth and have available ground-based observational data, only the errors in Δv due to the propulsion system will be considered, i.e. ϵ_M and ϵ_D from Table II. These σ values appear in the computer program as the constants INJVX and INJDV respectively.

In the following chapter, both the corrective program of this paper and the program of Lawden will be simulated on the computer. The same values of error will be used for each case. The results of these simulations should, therefore, be comparable to one another. In Chapters VI and VII the results will be presented graphically and discussed.

CHAPTER V

DISCUSSION OF THE COMPUTER SIMULATION

5.1 General

The computational method used in the digital computer program is that discussed in Chapter III. The computer utilized was an IBM 650 with peripheral equipment. Owing to the simplified nature of the computations, a minimum of computer time is required. The flow diagram, SOAP and machine language programs, and samples of input and output data are included (respectively for the program of this paper) in Appendixes C, D, and E hereto.

5.2 The Computer Program

The flow diagram of Appendix C is self-explanatory. A square root sub-routine was utilized in the program and is mentioned here and in the program only as the entry location 0031. An interesting feature of the program in Appendix D is the generation of the random normal numbers used for the Monte Carlo error simulation. The method utilizes the central-limit theorem and was developed in a paper by Fabrycky (14). The generation of the numbers is completed in machine language instructions 17 through 34. Variables generated by this method are distributed with $\mu = 0$ and $\sigma = 1$. These numbers are converted to the desired distribution by Equation 18.

$$18. \quad t_{x,y} = t_{0,1}(\sigma_{x,y}) + \mu_{x,y},$$

where

$t_{x,y}$ is the variable of interest,

$t_{0,1}$ is the variable developed, and

$\sigma_{x,y}, \mu_{x,y}$ relate to the distribution of interest.

Instructions 39 through 73 of the program convert the generated numbers to the correct σ values for ϵ_M and ϵ_D. The program as set up would be useful for a similar simulation of a trajectory with any desirable reference velocity, initial impulse or injection velocity, and time duration. It would only be necessary to enter the appropriate data on the sample input cards as shown in Appendix E. In order to modify the error distributions, it would be necessary to change SIGVX, SIGDV, INJVX, and INJDV to the desired values. Monte Carlo methods allow one to approach the assumed characteristics of the simulated population as the number of cycles or repetitions of the simulation increases. One of the more serious disadvantages to a simulation requiring the generation of a large volume of random normal deviates such as used here is the amount of computer time involved. By the generation method utilized here, approximately 135 milliseconds were required to develop each deviate on the IBM 650 computer.

The program listing of Appendix D is the simulation of the "practical" correction program in which corrections are made on or after $\tau = \tau_L$ as discussed in Chapter III. The listing of the theoretical program is very similar. In general, instructions 74 through 180 apply to both programs. However, when it is desired to run the theoretical program, branching instructions 93 and 123 should be removed as they cause the first correction to be made at τ_L. The data address on instruction 120 should also be changed to START. Owing to the similarity of these two programs only the "practical" one is reproduced. However the simulation of the Lawden program with input and output data is shown on the listing of Appendix F. Essentially the Lawden program is the same as the others except that the data input includes values of τ_i which are used in the calculation of the amount of correction.

5.3 Data Input and Output

Samples of the data output from each program are shown in Appendixes E and F and are labeled so as to be self-explanatory. Line one of the output data for each program is a reproduction of the data input card. For both the practical and theoretical programs, word four of line one is the value of Δv used in that set of calculations. In line two the first word appearing is a fixed point number recording the total number of times that a correction was made of the magnitude Δv. The second word of line two on the practical program records the total magnitude of all initial corrections made at τ_L and the third word is the total magnitude of all final corrections made at τ = one day. The average TC for some value of Δv would be the total of word one times Δv plus word two plus word three divided by the number of trips simulated as shown by Equation 18.

$$18. \qquad TC\ (\text{Practical}) = \frac{W_1(\Delta v) + W_2 + W_3}{n} \quad .$$

In line two of the theoretical program, word two is the total magnitude of the corrections made at τ = one day. Therefore the total cost for the theoretical program is

$$19. \qquad TC\ (\text{Theoretical}) = \frac{W_1(\Delta v) + W_2}{n} \quad .$$

In line three, each word which appears is the sum of the values of τ_i which resulted from the simulation. The word immediately below records the number of times the (i)th correction was made and below that is the average value of τ_i. The first word of lines three, four, and five are for $i = 1$, the second for $i = 2$, and so on.

On the Lawden simulation, the listings of which are in Appendix F, line two of the output contains, in order, the values of τ_i beginning with (i) equal to one. As stated previously, these values are calculated in Appendix B. Below each value of τ_i is the average value of the (i)th correction over "n" trips. Therefore, for the total cost one obtains,

$$20. \qquad TC\ (\text{Lawden}) = W_1 + W_2 \ldots + W_6 \ .$$

It would not be proper to end the discussion of the computer simulations without some mention of the shortcomings of them. In Chapter IV it was stated that the OA approach is usually begun by working with a comparatively simple model which is refined progressively until the required accuracy is achieved. So is it with the related computer simulation. The computer simulation behaves exactly as it is programmed to behave and, depending on its degree of refinement, develops output data more or less related to the associated mathematical model. From inspection of the output data of Appendix E, it can be seen that, in many cases, on the practical and theoretical programs it is not easy to determine how many corrections were made between τ_1 and τ_F since not all n trips required an intermediate correction. Though this can always be determined if it is remembered that in all cases there are n corrections made at τ_F = one day, it would be helpful if the computer program were improved. It had been anticipated that more frequent corrections would be required between τ_1 and τ_F. However, the computer simulation program is not at fault in this respect. The error, if any, lies in the assumed values for the error distribution parameters SIGDV, SIGVX, INJDV, and INJVX. The value of n used in this study was fifty. A larger n would have given smoother data at a corresponding increase in computer time required for the simulation.

CHAPTER VI

OUTPUT OF THE COMPUTER SIMULATION

6.1 Total Cost of Correction

The principal purpose of this study was stated earlier as being the application of OA methods toward optimizing the midcourse correction of deviations from an interplanetary reference trajectory. Of prime importance, therefore, is the presentation of data which will either substantiate or refute the realization of such a goal. In Fig's. 3 and 4 the average values of TC for each considered correction policy (Δv magnitude) are plotted for the theroetical program and the practical program respectively.

The theoretical program was simulated initially in order to observe if TC tended toward a minimum value for some particular value of Δv. The results given in Fig. 3 show that TC does tend toward a minimum value as $\Delta v \rightarrow 0$. Though the computer simulation calculated values of TC for Δv ranging from a minimum of one foot per second to a maximum of three hundred feet per second, only those obtained from Δv ranging from one foot per second to one hundred feet per second are plotted. Above the latter value of Δv, the relation of TC to Δv was approximately linear with the plot being a straight line extension of Fig's. 3 and 4.

Figures 3 and 4 are essentially identical above Δv of forty feet per second. At that value of Δv, all first corrections are made at $\tau_1 < \tau_L$ on both the theoretical and practical programs so this result is expected. Below forty feet per second, TC for the theoretical program tends toward an apparent minimum of zero while for the practical program TC appears to have a minimum slightly more than that for a Δv of one foot per second. This would be equivalent to making all corrections at $\tau_1 = \tau_L$ and τ_2 = one day with no intermediate corrections. Due to flattening of the TC curve, however, it is evident that a very slight difference is involved. The case of TC $\rightarrow$ zero for the theoretical program would occur as Δv was made smaller than one foot per second; however, at Δv = zero the computational method would be discontinuous.

6.2 Result of Delayed Correction

In Fig. 5 the relationship of TC to the average quantity $\tau_L - \tau_1$ (Time before first correction is made) is shown. The larger values of $\tau_L - \tau_1$ occur as Δv is increased which permits larger deviations to be uncorrected at τ_L. At $\tau_L - \tau_1 = \tau_L$, TC becomes infinite since this would imply that all correction must be made at $\tau = 0$. As discussed in Chapter III this event was not permitted in the simulation in order to avoid the discontinuity.

For clarity in the discussion, values of $\tau_L - \tau_1$ expressed in days are also shown on Fig. 5. What are, perhaps, the most useful conclusions to be drawn from this study are apparent from a study of this particular data plot. Since in reality the initial correction cannot be made at $\tau_1 = \tau_L$, the cost in propulsive energy of delaying the first correction can be read directly from this figure. In Chapter VII a discussion of these results will be presented.

6.3 General

The schedules of corrections for the Lawden (11) program and the Δv = one foot per second program are shown in Fig. 6. The former is an exponential curve the coordinates of which are calculated in Appendix B. In Fig. 7 is shown the total number of corrections for the simulation versus the magnitude of Δv. This information would permit one to consider a fixed cost of correction in the analysis if desired.

Tables IV, V, and VI give the numerical data plotted in Fig's. 3 through 7. Results and conclusions will be discussed in the following chapter. No great amount of data has been generated; however, the objective has not been to do so. From the information presented in this chapter it will be shown that the OA approach to this optimization problem is a useful one.

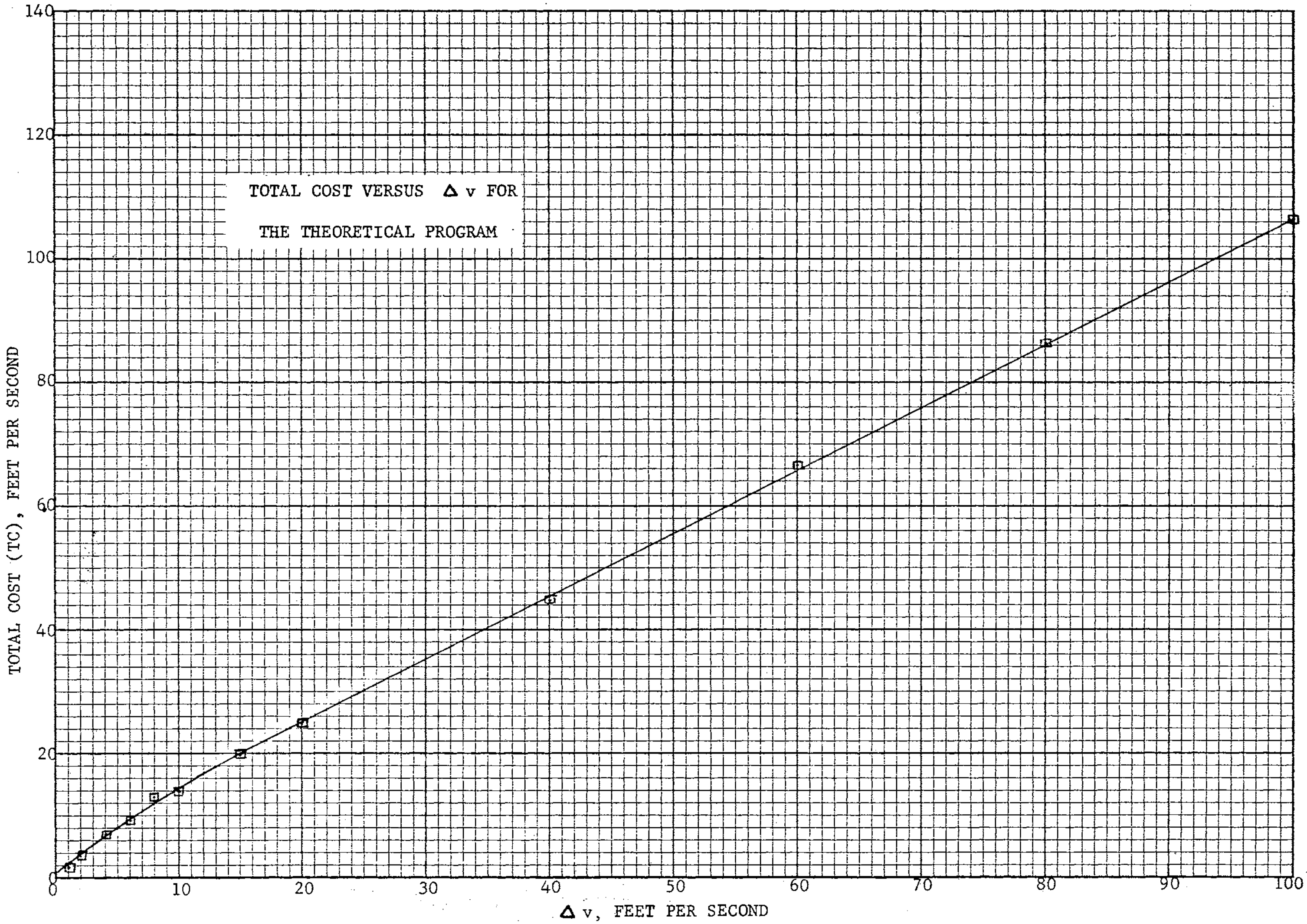
TOTAL COST VERSUS Δ v FOR
THE THEORETICAL PROGRAM
TOTAL COST (TC), FEET PER SECOND
Δ v, FEET PER SECOND
0
20
40
60
80
100
120
140
0
10
20
30
40
50
60
70
80
90
100

Fig. 3

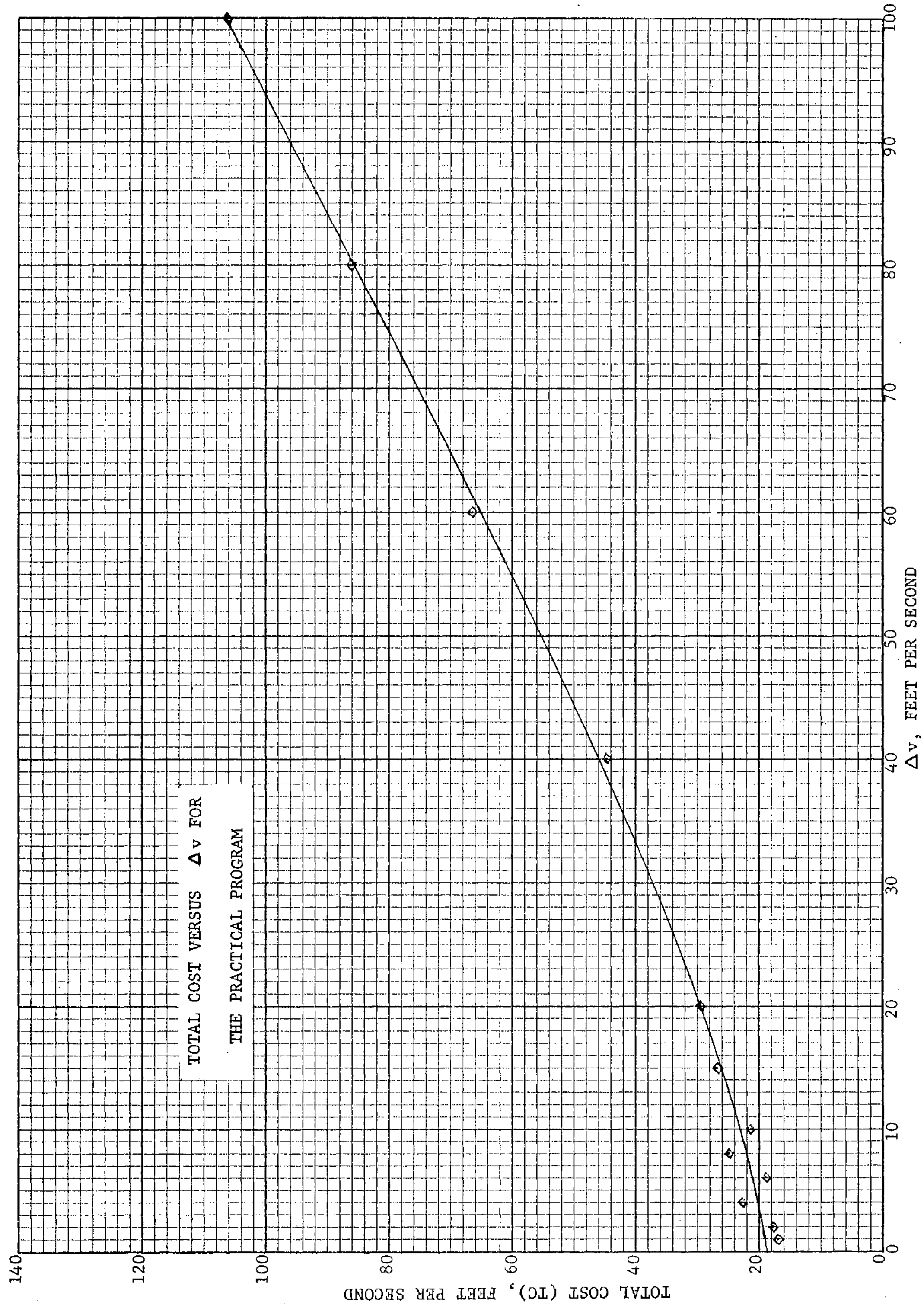
TOTAL COST VERSUS Δv FOR
THE PRACTICAL PROGRAM
Δv, FEET PER SECOND
TOTAL COST (TC), FEET PER SECOND
0
10
20
30
40
50
60
70
80
90
100
0
20
40
60
80
100
120
140

Fig. 4

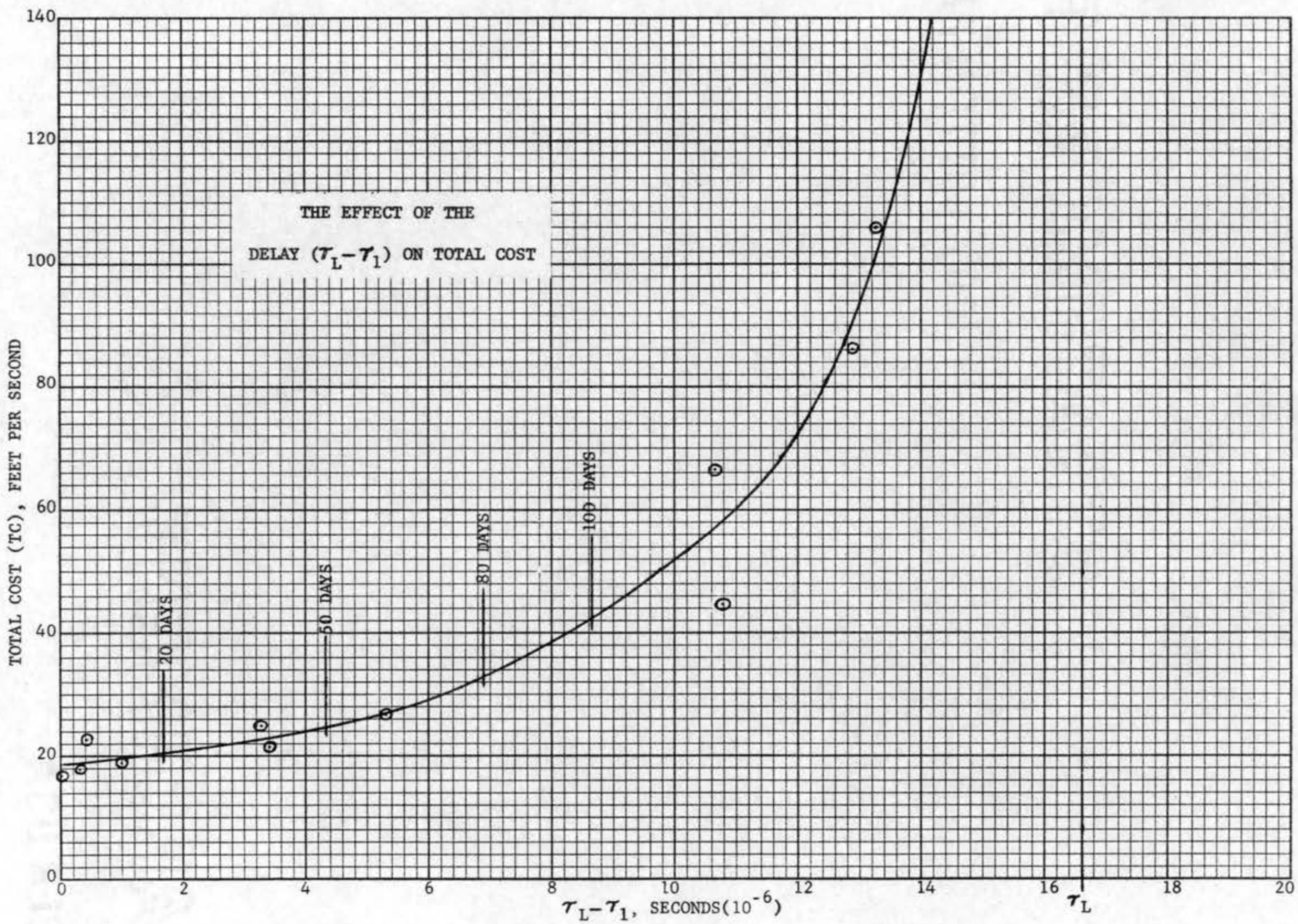
THE EFFECT OF THE
DELAY ($\tau_L - \tau_1$) ON TOTAL COST
TOTAL COST (TC), FEET PER SECOND
$\tau_L - \tau_1$, SECONDS(10^{-6})
τ_L
20 DAYS
50 DAYS
80 DAYS
100 DAYS
0
20
40
60
80
100
120
140
2
4
6
8
10
12
14
16
18
20

Fig. 5

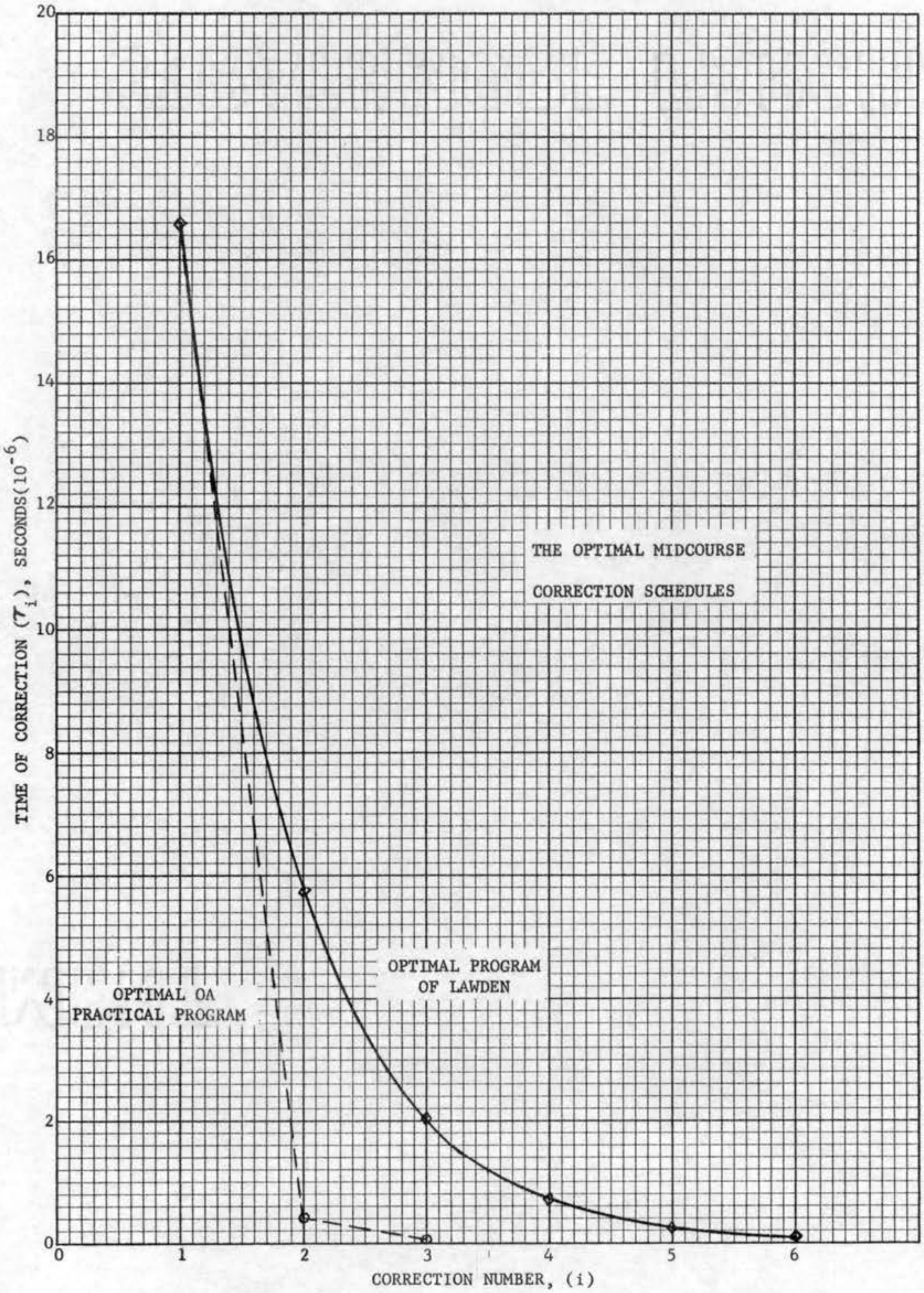
THE OPTIMAL MIDCOURSE
CORRECTION SCHEDULES
OPTIMAL PROGRAM
OF LAWDEN
OPTIMAL OA
PRACTICAL PROGRAM
TIME OF CORRECTION (τ_i), SECONDS(10^{-6})
CORRECTION NUMBER, (i)
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6

Fig. 6

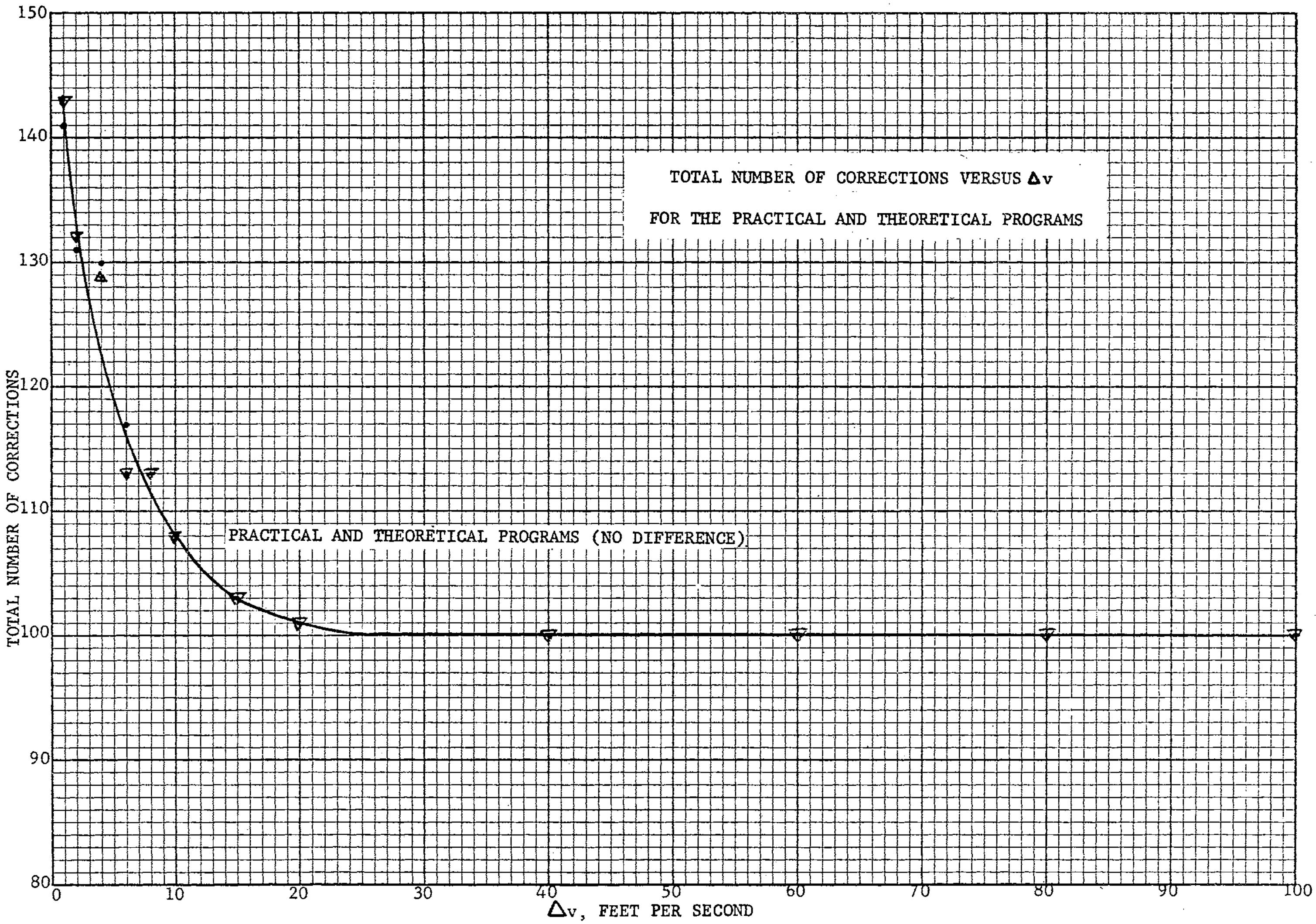
TOTAL NUMBER OF CORRECTIONS VERSUS Δv
FOR THE PRACTICAL AND THEORETICAL PROGRAMS
PRACTICAL AND THEORETICAL PROGRAMS (NO DIFFERENCE)
TOTAL NUMBER OF CORRECTIONS
Δv, FEET PER SECOND
150
140
130
120
110
100
90
80
0
10
20
30
40
50
60
70
80
90
100

Fig. 7

TABLE IV

TABULATION OF TC

Δv (feet/sec)	Theoretical TC (feet/sec)	Practical TC (feet/sec)
1	1.915	17.081
2	3.570	17.910
4	7.001	22.782
6	9.210	18.887
8	13.045	25.025
10	13.983	21.491
15	19.963	26.745
20	25.000	27.767
40	44.732	44.677
60	66.350	66.441
80	86.218	86.186
100	106.349	106.333
150	154.230	155.013
200	200.468	204.974
250	253.563	251.296
300	303.810	304.307

TABLE V

AVERAGE VALUE OF $\tau_L - \tau_1$

Δv (feet/sec)	Theoretical (sec x 10^{-6})	Practical (sec x 10^{-6})
1	-294.112	0
2	-118.063	0.305
4	- 65.916	0.419
6	- 25.307	1.512
8	- 22.668	0.996
10	- 9.776	3.285
15	- 3.615	3.410
20	+ 2.912	5.285
40	10.484	10.763
60	10.827	10.687
80	12.944	12.896
100	13.251	13.259
150	14.943	14.796
200	15.046	15.090
250	15.640	15.407
300	15.753	15.549

TABLE VI

TOTAL NUMBER OF CORRECTIONS MADE

Δv (feet /sec)	Theoretical Program	Practical Program
1	141	143
2	131	132
4	130	129
6	117	113
8	113	113
10	108	108
15	103	103
20	101	101
40	100	100
60	100	100
80	100	100
100	100	100
150	100	100
200	100	100
250	100	100
300	100	100

CHAPTER VII

RESULTS AND CONCLUSIONS

The problem of optimizing a program of correction for midcourse trajectory control was solved by Lawden (11) using the method of variational calculus. Operations Analysis (OA) has become a widely used analytical tool in the field of industrial engineering and has been utilized in studies of process optimization (10). In this study, the method of OA has been applied to the development of an optimal program for midcourse trajectory correction. It was expected that this approach would provide new insight into the problem not formerly available from the more classical treatment of Lawden.

The basis for this optimism is explained by a consideration of the limitations of the variational calculus method. In general, this method requires constrained initial and final conditions. The constraints are arbitrary and reveal little of the nature of the problem under study. In the problem of this paper, the initial constraint consists of the time of injection of the space vehicle into its reference trajectory and its location. The final constraint due to a fixed time of arrival and the required proximity of the target is essential to the analysis. In the variational approach, certain mathematical difficulties were encountered by

Lawden which prohibited a continuous solution unless a predetermined time of the first correction was established. Because the OA approach is not overly restricted by considerations of mathematical continuity, however, some of the difficulties inherent to the variational method can be avoided.

The results of this study demonstrate that:

a) Under the same restrictions imposed on the variational analysis, the results of the OA method agree with the results of that method.

b) With only the constraint that the final correction be made at τ = one day, the OA results are comparable to those of the variational method.

c) The OA method, further, demonstrates that the optimum corrective velocity increment is the minimum increment.

d) If a small corrective velocity increment is utilized, a provision for a larger initial increment is necessary.

e) The OA method can be used to determine the magnitude of the initial increment.

f) By the OA method, it is clear that the initial correction should be made as soon after injection as possible for optimization.

g) Lastly, it is probable that the OA approach can be applied to the solution of the more complicated problem of optimization on the basis of momentum correction rather than velocity correction.

By the variational method, a schedule of corrections of random magnitude is developed and plotted in Fig. 6. The computations of this schedule for the reference trajectory are shown in Appendix B. The time interval between succeeding corrections decreases in a geometric ratio. For the reference case, this method required three more corrections per trip than the program developed by the OA method. For comparison purposes, a practical program was developed in this paper utilizing the same initial and final constraints as the variational method, i.e. $\tau_1 \leqq \tau_L$, τ_F = one day. For a correctional velocity increment (Δv) of one foot per second, the average total cost (TC) of correction for this program was 17.0805 feet per second. For the variational method, the value of TC was 17.9082 as shown in Table VII.

TABLE VII

RESULTS OF LAWDEN SIMULATION

Correction (i)	τ_i (sec)	Δv_i (feet / sec)
1	$16.606(10^6)$	17.8189
2	$5.787(10^6)$	0.0885
3	$2.023(10^6)$	0.0007
4	$7.076(10^5)$	0.0001
5	$2.471(10^5)$	negligible
6	$8.640(10^4)$	negligible
	Total Δv	17.9082

From Figs. 3 and 4 it is seen that the values of TC for the OA programs decrease with the magnitude of the corrective impulse. This implies that the smallest possible correction should be made which, in turn, implies that it should occur as soon as possible after error occurs in order to reduce the time-cumulative effect of the error. The time-cumulative effect of injection error on TC is shown graphically in Fig. 5. For the ideal case, however, the theoretical OA program indicates, as seen in Table IV, that a correction should be made before injection or before the initial error has occurred. By imposing the constraint that the first correction be made at the moment of injection, it is possible on comparison of Figs. 3 and 4 to predict the initial correction cost. A comparison of these results also gives an indication of the added correction cost due to an absence of pre-injection corrections. This cost of delay can also be inferred from the fact that for the larger values of Δv resulting in a later first correction, the value of TC is greater.

As applied to the study of optimal inventory policy, the flexibility of the OA approach permits one to consider both variable and fixed costs. An appreciation of this flexibility can be gained by observation of the small changes in the computer simulation required to modify the theoretical program to the practical program. The versatility demonstrated by the OA method encourages the author in the feeling that refinement of the program of this paper to consider momentum expenditure rather than velocity expenditure is a logical step. It should, further, not be difficult to include the consideration of fixed costs of correction to the analysis. Fixed cost of correction varies in proportion to the number of corrections made instead of the magnitude of the correction. The information gained from this study would indicate that fixed costs tend toward a minimum for a non-zero value of Δv . These results are shown in Fig. 7. If a minimum cost (optimal) correction program does exist under these additional conditions, the OA method provides promise of defining it.

In general, a primary consideration in the design of space vehicles is the minimization of the injected weight to permit maximum utilization of propulsive energy. Part of the weight minimization problem is the optimization of the expenditure of corrective fuel. By use of the variational method, Lawden developed an optimal program which was subject to limiting restrictions. The OA method has permitted an analysis without these restrictions. At the minimum, the OA approach has provided results comparable to those of the variational method; it has yielded more useful information about the problem; and, finally, the OA method appears to provide the means for a more realistic analysis of the problem of optimization.

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APPENDIX A

CALCULATION OF REFERENCE TRAJECTORY PARAMETERS

A.1 Calculation of Approximate Trajectory Length

$\theta_E = 29.5^o \; (\tau = \tau_L)$

$\theta_M = 141.50^o \; (\tau = 0)$

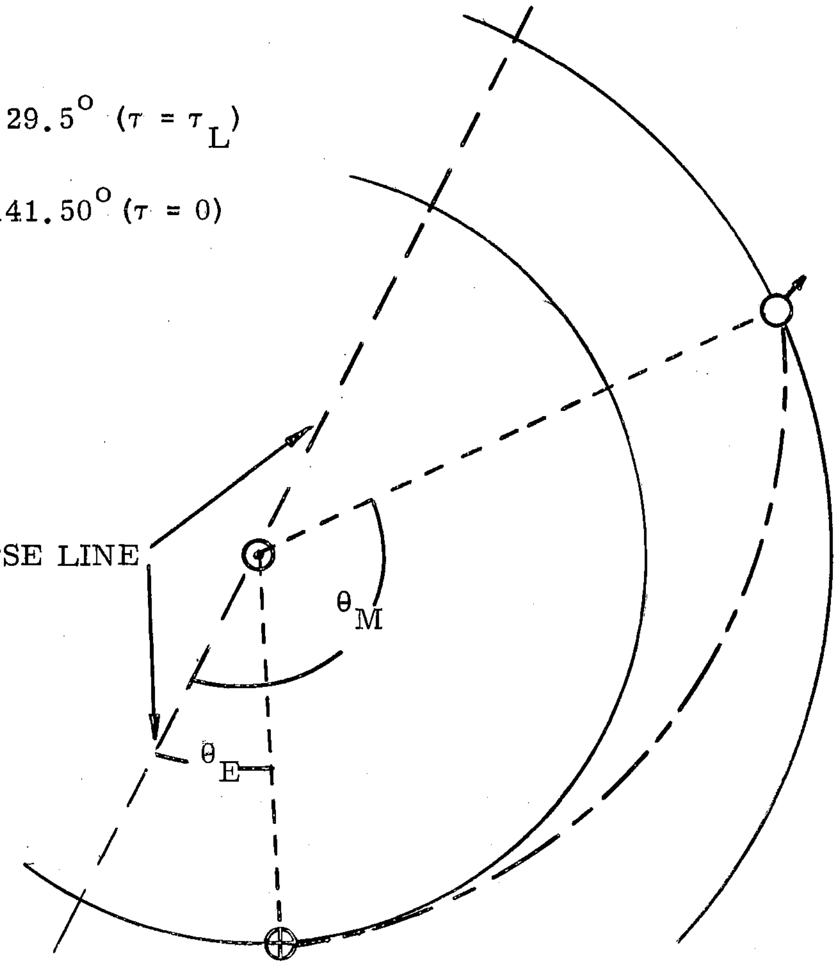


Fig. A.1 The Transfer Trajectory

A.2 Approximate Orbital and Trajectory Parameters (15)

Semi-Major Axis (a):	Earth Orbit (a_E)	= 1.000 A.U.
	Mars Orbit (a_M)	= 1.524 A.U.
	Reference Trajectory (a_T)	= 1.306 A.U.
Eccentricity (e):	Reference Trajectory (e_T)	= 0.254

A.3 Calculation of θ_E at $\tau = \tau_L$

$$\theta_E \cong \cos^{-1} \frac{a_T(1 - e_T^2) - a_E}{a_E e_T}$$

$$\cong \text{Cos}^{-1} \frac{1.306(1 - 0.254^2) - 1.000}{1.000\,(0.254)}$$

$$\cong 29.5^\circ$$

A. 4 Calculation of θ_M at $\tau = \tau_F$

$$\theta_m \cong \text{Cos}^{-1} \frac{a_T(1 - e_T^2) - a_M}{a_m e_T}$$

$$\cong \text{Cos}^{-1} \frac{1.306(1 - 0.254^2) - 1.524}{1.524\,(0.254)}$$

$$\cong 141.5^\circ$$

A. 5 Calculation of Arc Length[1]

$\phi_1 = 41.10^\circ$

$\phi_2 = 67.08^\circ$

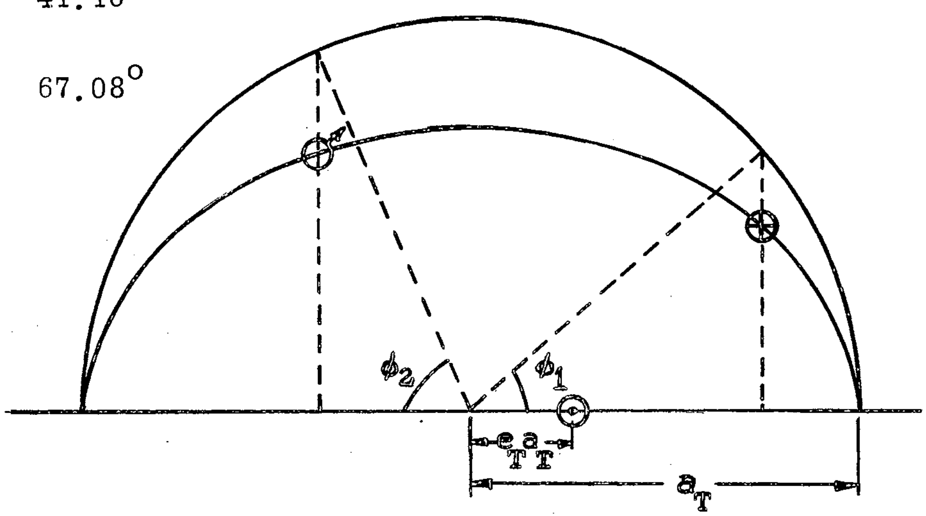


Fig. A. 2 Transfer Ellipse Parameters

$E_{\phi_1} = 1.306\,(0.7173) = 0.9368$ A.U.

$E_{\phi_2} = 1.306\,(1.1575) = 1.5117$ A.U.

---

[1]Calculated from interpolated elliptic integral values from ref. (16).

$$E_{180^o} = 1.306\,(3.0902) = 4.0358 \text{ A.U.}$$

$$E_{\phi_1\phi_2} = 4.0358 - (0.9368 + 1.5117) = 1.5873 \text{ A.U.}$$

$$= 7.7943\,(10^{11}) \text{ feet}$$

$$v_L = \frac{7.7943\,(10^{11})}{192.2\,(86,400)}$$

$$= 46,936 \text{ feet/second}$$

APPENDIX B

NUMERICAL CALCULATION OF THE LAWDEN SCHEDULE

B.1 Calculation of Parameters for Lawden (11) Program

$$\alpha = \frac{\tau_L}{\tau_F}$$

$$= \frac{192.2}{1.0}$$

$$= 192.2 \text{ days}$$

$$n \doteq \ell n \alpha + 1$$

$$\doteq \ell n (192.2) + 1$$

$$\doteq 6.25 \quad \text{, we take } n = 6 \text{ ,}$$

$$R = \alpha^{1/n-1}$$

$$= (192.2)^{1/5}$$

$$= 2.86$$

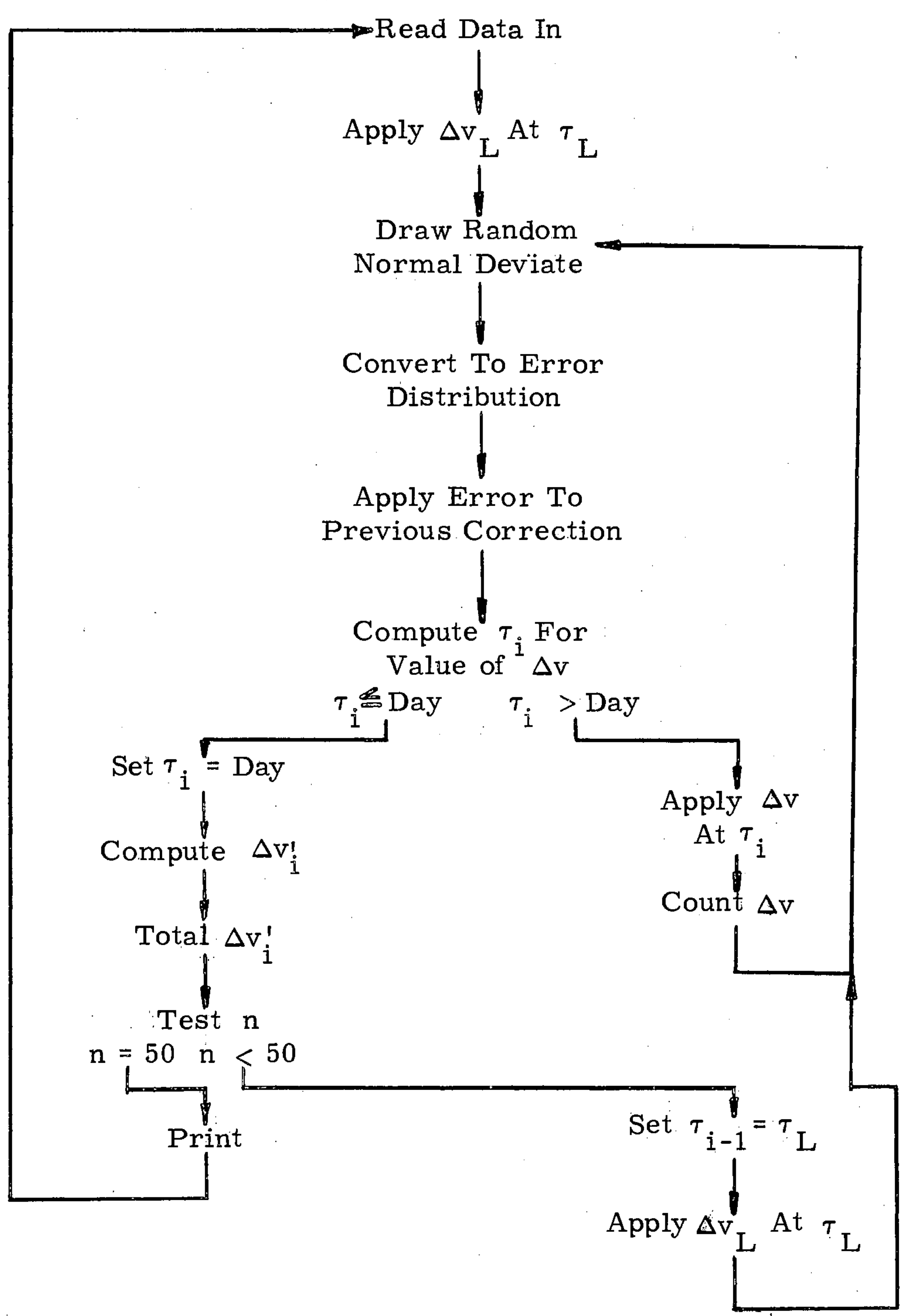
$$\tau_1 = \tau_L R^{-i+1} \quad \text{, where } \tau_1 = \tau_L \text{ , } \tau_F = \tau_6 \text{ .}$$

τ_1	=	192.2 days			=	$1.6606 (10^7)$ sec.
τ_2	=	$192.2 (2.86)^{-1}$	=	66.98 days	=	$5.7871 (10^6)$ sec.
τ_3	=	$192.2 (2.86)^{-2}$	=	23.42 days	=	$2.0235 (10^6)$ sec.
τ_4	=	$192.2 (2.86)^{-3}$	=	8.19 days	=	$7.0762 (10^5)$ sec.
τ_5	=	$192.2 (2.86)^{-4}$	=	2.86 days	=	$2.4710 (10^5)$ sec.
τ_6	=	$192.2 (2.86)^{-5}$	=	1.00 days	=	$8.6400 (10^5)$ sec.

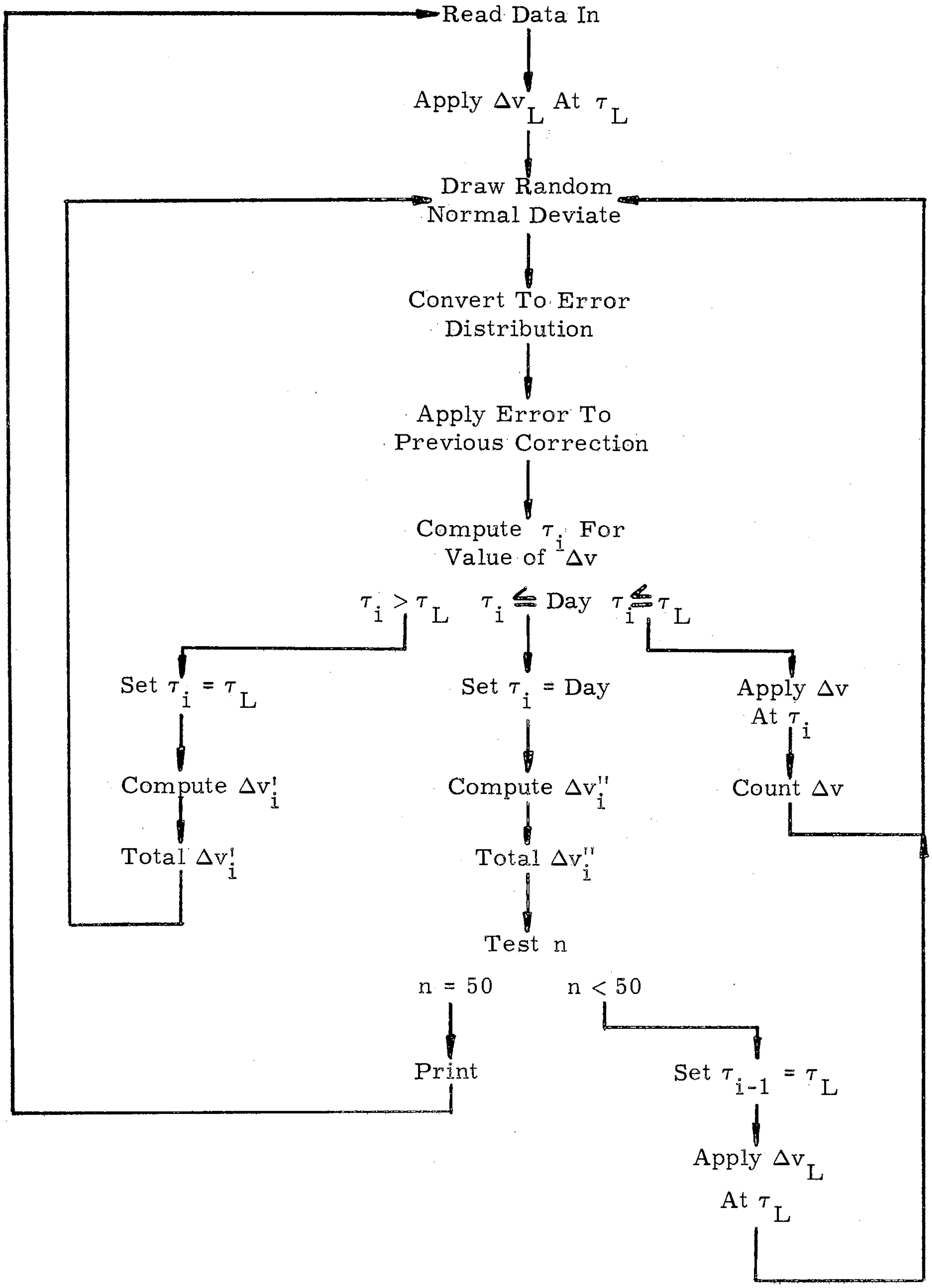
APPENDIX C

SIMULATION FLOW DIAGRAMS

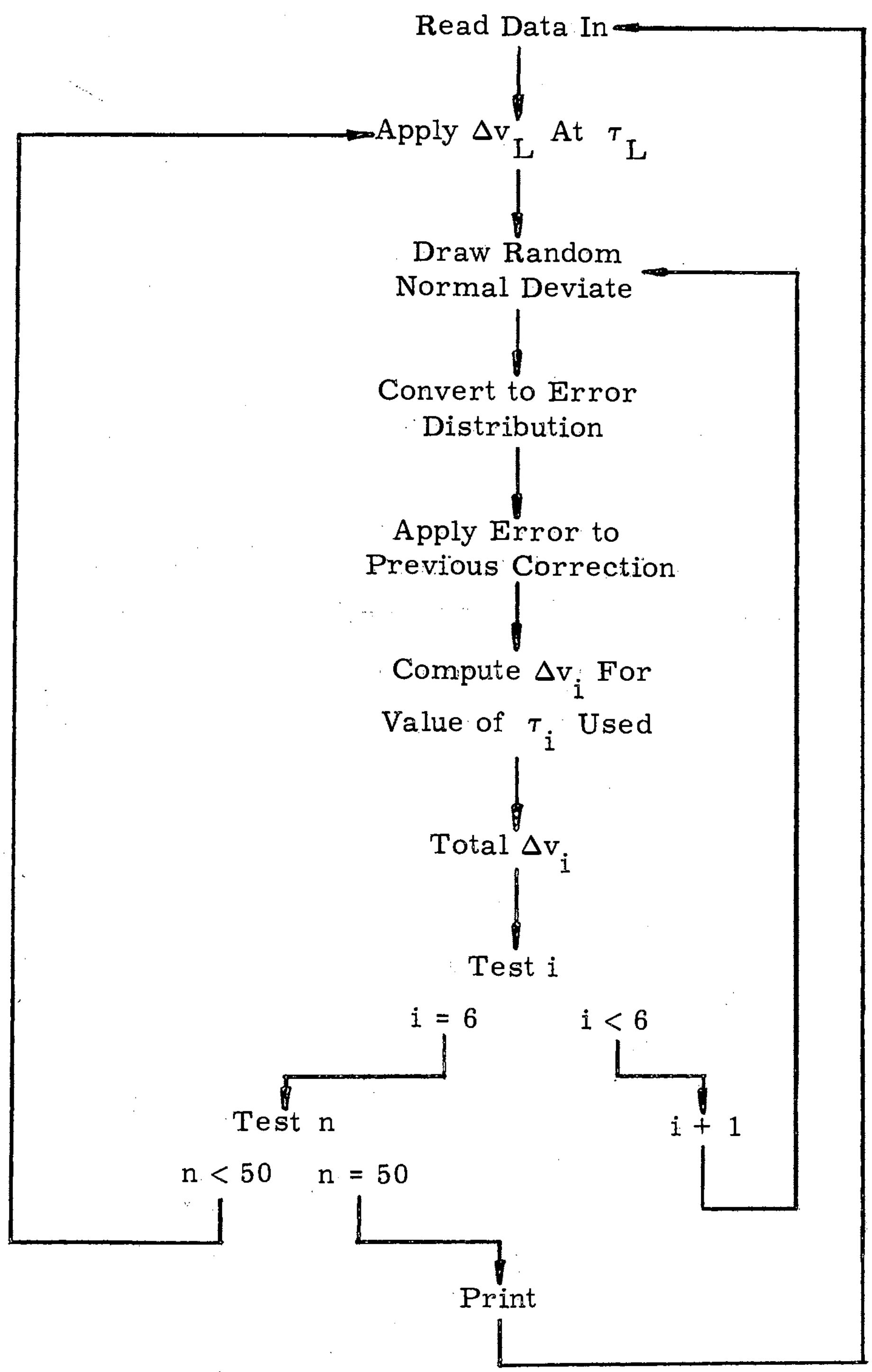
FLOW DIAGRAM - THEORETICAL PROGRAM



FLOW DIAGRAM - PRACTICAL PROGRAM



FLOW DIAGRAM - LAWDEN PROGRAM



APPENDIX D

OA PRACTICAL PROGRAM COMPUTER SIMULATION

1						BLR	0000	0100
2	0150	82	0000	0106	READ	RAB	0000	
3	0106	88	0000	0112		RAC	0000	
4	0112	69	0115	0118		LDD	ZERO	
5	0118	24	0121	0124		STD	RPT	
6	0124	24	0127	0130		STD	INCR	
7	0130	24	0133	0136		STD	PART	
8	0136	24	0139	0142		STD	TOTAL	SETUP
9	0142	69	0115	0168	SETUP	LDD	ZERO	
10	0168	24	9420	0123		STD	9420	
11	0123	53	0039	0129		SXB	0039	
12	0129	42	0132	0183		NZB		RAB
13	0132	52	0040	0142		AXB	0040	SETUP
14	0183	82	0000	0189	RAB	RAB	0000	
15	0189	70	9016	0239		RD1	9016	
16	0239	74	9016	0289		WR2	9016	START
17	0289	60	8002	0147	START	RAU	8002	
18	0147	20	0101	0104		STL	KEEP	
19	0104	80	0015	0110		RAA	0015	LOOPA
20	0110	60	0113	0117	LOOPA	RAU	RANDM	
21	0117	19	0120	0140		MPY	ODD	
22	0140	20	0120	0173		STL	ODD	
23	0173	65	8002	0131		RAL	8002	
24	0131	30	0006	0145		SRT	0006	
25	0145	15	0101	0105		ALO	KEEP	
26	0105	20	0101	0154		STL	KEEP	
27	0154	51	0001	0160		SXA	0001	
28	0160	40	0110	0114		NZA	LOOPA	
29	0114	60	0101	0155		RAU	KEEP	
30	0155	19	0108	0128		MPY	CONE	
31	0128	16	0181	0135		SLO	CTWO	
32	0135	65	8002	0143		RAL	8002	
33	0143	31	0001	0149		SRD	0001	
34	0149	35	0002	0205		SLT	0002	
35	0205	46	0158	0109		BMI	YES	NO
36	0158	16	0111	0165	YES	SLO	FLP	PRO
37	0109	15	0111	0165	NO	ALO	FLP	PRO
38	0165	32	8002	0195	PRO	FAD	8002	
39	0195	42	0148	0199		NZB		INIT
40	0148	48	0151	0102		NZC	RDV	RVX
41	0199	48	0152	0103	INIT	NZC	IRDV	IRVX
42	0103	20	9052	0210	IRVX	STL	9052	
43	0210	39	0163	0213		FMP	INJVX	
44	0213	39	9018	0116		FMP	9018	
45	0116	32	9017	0245		FAD	9017	
46	0245	24	9055	0201		STD	9055	
47	0201	39	8003	0255		FMP	8003	
48	0255	21	0260	0263		STU	VXVX	
49	0263	69	9016	0119		LDD	9016	
50	0119	24	9053	0125		STD	9053	
51	0125	24	9001	0137		STD	9001	
53	0137	88	0001	0289		RAC	0001	START
54	0152	39	0305	0355	IRDV	FMP	INJDV	

```
0355   39 9018 0208             FMP    9018
0208   21 0162 0215             STU    DV
0215   39 8003 0169             FMP    8003
0169   21 0174 0177             STU    DVDV
0177   88 0000 0233             RAC    0000
0233   82 0001 0339             RAB    0001     CALC
0102   39 0405 0455    RVX      FMP    SIGVX
0455   39 9002 0258             FMP    9002
0258   32 9055 0187             FAD    9055
0187   39 8003 0141             FMP    8003
0141   21 0260 0313             STU    VXVX
0313   88 0001 0289             RAC    0001     START
0151   39 0204 0254    RDV      FMP    SIGDV
0254   39 9002 0107             FMP    9002
0107   21 0162 0265             STU    DV
0265   39 8003 0219             FMP    8003
0219   21 0174 0227             STU    DVDV
0227   88 0000 0283             RAC    0000
0283   52 0001 0339             AXB    0001     CALC
0339   60 0260 0315    CALC     RAU    VXVX
0315   33 0174 0251             FSB    DVDV
0251   69 0304 0031             LDD              0031
0304   21 0308 0161             STU    VEESR
0161   60 9055 0269             RAU    9055
0269   33 0308 0185             FSB    VEESR
0185   21 0190 0193             STU    VEE
0193   39 8003 0197             FMP    8003
0197   32 0174 0301             FAD    DVDV
0301   69 0354 0031             LDD              0031
0354   39 9053 0157             FMP    9053
0157   24 9001 0363             STD    9001
0363   34 9019 0166             FDV    9019
0166   24 9002 0122             STD    9002     TAUA
0122   21 9053 0179    TAUA     STU    9053
0179   33 0182 0159             FSB    DAY
0159   46 0212 0413             BMI    FINAL
0413   60 9001 0171             RAU    9001
0171   33 9053 0351             FSB    9053
0351   46 0404 0505             BMI    TAUB
0505   60 9053 0463             RAU    9053
0463   32 9429 0555             FAD    9429
0555   21 9429 0232             STU    9429
0232   60 0235 0389             RAU    ONEFL
0389   32 9419 0295             FAD    9419
0295   21 9419 0172             STU    9419
0172   60 9001 0229             RAU    9001
0229   34 9053 0282             FDV    9053     CORR
0282   21 0186 0439    CORR     STU    RTAU
0439   39 0190 0240             FMP    VEE
0240   21 0144 0247             STU    DVY
0247   60 0186 0191             RAU    RTAU
0191   39 0162 0262             FMP    DV
0262   21 0216 0319             STU    DVZ
0319   60 0144 0249             RAU    DVY
```

```
0249   32 0308 0285                FAD  VEESR
0285   39 8003 0489                FMP   8003
0489   21 0194 0297                STU  CVYSQ
0297   60 0216 0221                RAU  DVZ
0221   33 0162 0539                FSB  DV
0539   39 8003 0243                FMP   8003
0243   32 0194 0271                FAD  CVYSQ
0271   69 0224 0031                LDD              0031
0224   21 9055 0281                STU   9055
0281   60 9053 0589                RAU   9053
0589   33 0182 0209                FSB  DAY
0209   44 0513 0164                NZU             LAST
0513   60 9053 0321                RAU   9053
0321   33 9016 0401                FSB   9016
0401   44 0289 0156                NZU  START      IMP
0164   60 8006 0371          LAST  RAU   8006
0371   10 0139 0293                AUP  TOTAL
0293   21 0139 0192                STU  TOTAL
0192   82 0000 0198                RAB   0000
0198   58 0001 0454                AXC   0001
0454   60 0121 0175                RAU  RPT
0175   32 0235 0211                FAD  ONEFL
0211   21 0121 0156                STU  RPT        IMP
0156   60 0235 0639          IMP   RAU  ONEFL
0639   30 0009 0259                SRT   0009
0259   10 0133 0237                AUP  PART
0237   21 0133 0236                STU  PART
0236   58 0001 0242                AXC   0001
0242   60 0144 0299                RAU  DVY
0299   39 8003 0153                FMP   8003
0153   21 0358 0261                STU  DVYSQ
0261   60 0216 0421                RAU  DVZ
0421   39 8003 0225                FMP   8003
0225   32 0358 0335                FAD  DVYSQ
0335   69 0138 0031                LDD              0031
0138   21 9002 0345                STU   9002
0345   32 9657 0333                FAD   9657
0333   21 9657 0310                STU   9657
0310   60 0563 0167                RAU  FIFFL
0167   33 0121 0347                FSB  RPT
0347   44 0451 0202                NZU             TAB
0451   88 0000 0289                RAC   0000      START
0202   60 0139 0343          TAB   RAU  TOTAL
0343   11 0133 0287                SUP  PART
0287   21 9057 0395                STU   9057
0395   74 9057 0445                WR2   9057
0445   74 9030 0495                WR2   9030
0495   74 9020 0545                WR2   9020      DIV
0545   60 9230 0385          DIV   RAU   9230
0385   34 9220 0170                FDV   9220
0170   21 9240 0393                STU   9240
0393   50 0001 0349                AXA   0001
0349   51 0010 0605                SXA   0010
0605   40 0408 0309                NZA             PRT
```

163	0408	50	0010	0545		AXA	0010	DIV
164	0309	74	9040	0150	PRT	WR2	9040	READ
165	0212	60	0182	0122	FINAL	RAU	DAY	TAUA
166	0404	60	9016	0122	TAUB	RAU	9016	TAUA
167	0108	00	0000	8944	CONE	00	0000	8944
168	0181	06	7082	0000	CTWO	06	7082	0000
169	0204	20	0000	0047	SIGDV	20	0000	0047
170	0305	10	0000	0047	INJDV	10	0000	0047
171	0405	20	0000	0048	SIGVX	20	0000	0048
172	0163	10	0000	0048	INJVX	10	0000	0048
173	0182	86	4000	0055	DAY	86	4000	0055
174	0111	00	0000	0051	FLP	00	0000	0051
175	0113	00	0001	0101	RANDM	00	0001	0101
176	0120	12	3456	7700	ODD	12	3456	7700
177	115	00	0000	0000	ZERO	00	0000	0*
178	0200	49	0000	0052	COUNT	49	0000	0052
179	0235	10	0000	0051	ONEFL	10	0000	0051
180	0563	50	0000	0052	FIFFL	50	0000	0052

APPENDIX E

INPUT AND OUTPUT DATA

OF THE OA PROGRAM

I. THEORETICAL PROGRAM OUTPUT

Line	W1	W2	W3	W4
1	1660608058+	4693600055+	2193600055+	1000000051+
2	91+	4750355051+		
3	1553588561+	2418792858+	3542400057+	
4	5000000052+	5000000052+	4100000052+	
5	3107177059+	4837585656+	8640000055+	
1	1660608058+	4693600055+	2193600055+	2000000051+
2	81+	1650324952+		
3	6733455460+	1242755058+	2678400057+	
4	5000000052+	5000000052+	3100000052+	
5	1346691159+	2485510056+	8640000055+	
1	1660608058+	4693600055+	2193600055+	4000000051+
2	80+	3005521852+		
3	4126088860+	1019339658+	2592000057+	
4	5000000052+	5000000052+	3000000052+	
5	8252177658+	2038679256+	8640000055+	
1	1660608058+	4693600055+	2193600055+	6000000051+
2	67+	5848384252+		
3	2095670760+	5769450957+	1468800057+	
4	5000000052+	5000000052+	1700000052+	
5	4191341458+	1153890256+	8640000055+	

II. PRACTICAL PROGRAM OUTPUT

Line	W1	W2	W3	W4
1	1660608058	4693600055	2193600055	1000000051
2	43+	8059970953+	5028795851+	
3	8303040059+	2344433258+	3715200057+	
4	5000000052+	5000000052+	4300000052+	
5	1660608058+	4688866456+	8640000055+	
1	1660608058+	4693600055+	2193600055+	2000000051+
2	35+	8098943253+	1558018952+	
3	8150508159+	1260723958+	2764800057+	
4	5000000052+	5000000052+	3200000052+	
5	1630101658+	2521447856+	8640000055+	
1	1660608058+	4693600055+	2193600055+	4000000051+
2	32+	9779114053+	3317728852+	
3	8093423959+	9952681457+	2505600057+	
4	5000000052+	5000000052+	2900000052+	
5	1618684858+	1990536356+	8640000055+	
1	1660608058+	4693600055+	2193600055+	6000000051+
2	28+	7009409353+	7538727652+	
3	7546729259+	5452063657+	1123200057+	
4	5000000052+	5000000052+	1300000052+	
5	1509345858+	1090412756+	8640000055+	

Input Data

In Line 1: W1 = τ_L; W2 = v_L; W3 = Δv_L and W4 = Δv .

APPENDIX F

LAWDEN PROGRAM COMPUTER SIMULATION

1						BLR	0000	0100
2	0150	82	0000	0106	READ	RAB	0000	
3	0106	88	0000	0112		RAC	0000	
4	0112	69	0115	0118		LDD	ZERO	
5	0118	24	0121	0124		STD	RPT	
6	0124	24	0127	0130		STD	INCR	
7	0130	24	0133	0136		STD	PART	
8	0136	24	0139	0142		STD	TOTAL	SETUP
9	0142	69	0115	0168	SETUP	LDD	ZERO	
10	0168	24	9420	0123		STD	9420	
11	0123	53	0039	0129		SXB	0039	
12	0129	42	0132	0183		NZB		RAB
13	0132	52	0040	0142		AXB	0040	SETUP
14	0183	82	0000	0189	RAB	RAB	0000	
15	0189	70	9012	0239		RD1	9012	
16	0239	74	9012	0289		WR2	9012	START
17	0289	60	8002	0147	START	RAU	8002	
18	0147	20	0101	0104		STL	KEEP	
19	0104	80	0015	0110		RAA	0015	LOOPA
20	0110	60	0113	0117	LOOPA	RAU	RANDM	
21	0117	19	0120	0140		MPY	ODD	
22	0140	20	0120	0173		STL	ODD	
23	0173	65	8002	0131		RAL	8002	
24	0131	30	0006	0145		SRT	0006	
25	0145	15	0101	0105		ALO	KEEP	
26	0105	20	0101	0154		STL	KEEP	
27	0154	51	0001	0160		SXA	0001	
28	0160	40	0110	0114		NZA	LOOPA	
29	0114	60	0101	0155		RAU	KEEP	
30	0155	19	0108	0128		MPY	CONE	
31	0128	16	0181	0135		SLO	CTWO	
32	0135	65	8002	0143		RAL	8002	
33	0143	31	0001	0149		SRD	0001	
34	0149	35	0002	0205		SLT	0002	
35	0205	46	0158	0109		BMI	YES	NO
36	0158	16	0111	0165	YES	SLO	FLP	PRO
37	0109	15	0111	0165	NO	ALO	FLP	PRO
38	0165	32	8002	0195	PRO	FAD	8002	
39	0195	42	0148	0199		NZB		INIT
40	0148	48	0151	0102		NZC	RDV	RVX
41	0199	48	0152	0103	INIT	NZC	IRDV	IRVX
42	0103	20	9052	0210	IRVX	STL	9052	
43	0210	39	0163	0213		FMP	INJVX	
44	0213	39	9014	0116		FMP	9014	
45	0116	32	9013	0245		FAD	9013	
46	0245	24	9055	0201		STD	9055	
47	0201	39	8003	0255		FMP	8003	
48	0255	21	0260	0263		STU	VXVX	
49	0263	69	9012	0119		LDD	9012	
50	0119	24	9053	0125		STD	9053	
51	0125	24	9001	0231		STD	9001	
52	0231	24	9021	0137		STD	9021	
53	0137	88	0001	0289		RAC	0001	START
54	0152	39	0305	0355	IRDV	FMP	INJDV	

```
0355  39 9014 0208          FMP   9014
0208  21 0162 0215          STU   DV
0215  39 8003 0169          FMP   8003
0169  21 0174 0177          STU   DVDV
0177  88 0000 0233          AC    0000
0233  82 0001 0339          RAB   0001   CALC
0102  39 0405 0455   RVX    FMP   SIGVX
0455  39 9002 0258          FMP   9002
0258  32 9055 0187          FAD   9055
0187  39 8003 0141          FMP   8003
0141  21 0260 0313          STU   VXVX
0313  88 0001 0289          RAC   0001   START
0151  39 0204 0254   RDV    FMP   SIGDV
0254  39 9002 0107          FMP   9002
0107  21 0162 0265          STU   DV
0265  39 8003 0219          FMP   8003
0219  21 0174 0227          STU   DVDV
0227  88 0000 0283          RAC   0000
0283  52 0001 0339          AXB   0001   CALC
0339  60 0260 0315   CALC   RAU   VXVX
0315  33 0174 0251          FSB   DVDV
0251  69 0304 0031          LDD          0031
0304  21 0308 0161          STU   VEESR
0161  60 9055 0269          RAU   9055
0269  33 0308 0185          FSB   VEESR
0185  21 0190 0193          STU   VEE
0193  39 8003 0197          FMP   8003
0197  32 0174 0301          FAD   DVDV
0301  69 0354 0031          LDD          0031
0354  39 9001 0157          FMP   9001
0157  34 9053 0310          FDV   9053
0310  21 9002 0167          STU   9002
0167  60 9001 0175          RAU   9001
0175  34 9053 0178          FDV   9053
0178  24 9001 0134          STD   9001   CORR
0134  21 0138 0191   CORR   STU   RTAU
0191  39 0190 0240          FMP   VEE
0240  21 0144 0247          STU   DVY
0247  60 0138 0243          RAU   RTAU
0243  39 0162 0212          FMP   DV
0212  21 0166 0319          STU   DVZ
0319  60 0144 0249          RAU   DVY
0249  32 0308 0235          FAD   VEESR
0235  39 8003 0389          FMP   8003
0389  21 0194 0297          STU   CVYSQ
0297  60 0166 0171          RAU   DVZ
0171  33 0162 0439          FSB   DV
0439  39 8003 0293          FMP   8003
0293  32 0194 0221          FAD   CVYSQ
0221  69 0224 0031          LDD          0031
0224  21 9055 0281          STU   9055
0281  60 9430 0285          RAU   9430
0285  32 9002 0365          FAD   9002
0365  21 9430 0333          STU   9430
```

```
109 0333  53 0006 0489           SXB   0006
110 0489  42 0192 0343           NZB          FIN
111 0192  52 0006 0198           AXB   0006
112 0198  60 9414 0369           RAU   9414
113 0369  21 9053 0277           STU   9053
114 0277  21 9421 0289           STU   9421   START
115 0343  60 0133 0237   FIN     RAU   PART
116 0237  32 0290 0217           FAD   ONEFL
117 0217  21 0133 0186           STU   PART
118 0186  60 0539 0393           RAU   COUNT
119 0393  33 0133 0159           FSB   PART
120 0159  46 0262 0289           BMI   DIV    START
121 0262  60 9231 0335   DIV     RAU   9231
122 0335  34 0188 0238           FDV   FIFFL
123 0238  21 9231 0184           STU   9231
124 0184  51 0005 0340           SXA   0005
125 0340  40 0443 0244           NZA          WR
126 0443  50 0006 0262           AXA   0006   DIV
127 0244  74 9021 0294   WR      WR2   9021
128 0294  74 9031 0150           WR2   9031   READ
129 0108  00 0000 8944   CONE    00    0000   8944
130 0181  06 7082 0000   CTWO    06    7082   0000
131 0204  20 0000 0047   SIGDV   20    0000   0047
132 0305  10 0000 0047   INJDV   10    0000   0047
133 0405  20 0000 0048   SIGVX   20    0000   0048
134 0163  10 0000 0048   INJVX   10    0000   0048
135 0200  86 4000 0055   DAY     86    4000   0055
136 0111  00 0000 0051   FLP     00    0000   0051
137 0113  00 0001 0101   RANDM   00    0001   0101
138 0120  12 3456 7700   ODD     12    3456   7700
139  115  00 0000 0000   ZERO    00    0000   0**
140 0539  49 0000 0052   COUNT   49    0000   0052
141 0290  10 0000 0051   ONEFL   10    0000   0051
142 0188  50 0000 0052   FIFFL   50    0000   0052
```

LAWDEN PROGRAM OUTPUT

Line	W1	W2	W3	W4
1	1660608058	4693600055	2193600055	5787072057
2	1660608058+	5787072057+	2023488057+	7076160056+
3	1660608058+	5787072057+	2023488057+	7076160056+

Line	W5	W6	W7	W8
1	2023488057	7076160056	2471040056	8640000055
2	2471040056+	8640000055+		
3	2471040056+	8640000055+		

Input Data

In Line 1: W1 = τ_L; W2 = v_L; W3 = Δv_L; W4 = τ_2; W5 = τ_3; W6 = τ_4; W7 = τ_5; W8 = τ_6 .

VITA

James Robert Coffee

Candidate for the Degree of

Master of Science

Thesis: AN APPROACH TO THE DEVELOPMENT OF AN OPTIMAL PROGRAM FOR THE CORRECTION OF DEVIATIONS FROM AN INTERPLANETARY REFERENCE TRAJECTORY

Major Field: General Engineering

Biographical:

Personal Data: Born in Pawhuska, Oklahoma, September 9, 1928, the son of Floyd and Georgia E. Coffee.

Education: Attended public schools in Cushing, Oklahoma, graduating from Cushing High School in 1946. Received the Bachelor of Science degree from the Oklahoma State University in 1956 with a major in General Engineering. Entered the Graduate School of Oklahoma State University in September, 1961, and completed the requirements for the Master of Science degree in May, 1963.

Professional Experience: Employed from August, 1956, to December, 1958, as a production engineer in the Jackson, Mississippi, district office of The Humble Oil Company. Employed from December, 1958 to September, 1961, as a Ground Support Equipment designer and project engineer in the Wichita, Kansas, Division of The Boeing Company. From September, 1961 to May, 1963, have been employed as a part-time instructor in the Office of Engineering Research at Oklahoma State University.