Geometrical dependencies of Agaricaceae (FR.) COHN. Basidia – as an additional taxonomic criterion

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Introduction

The taxonomy of species, genera, families and orders of higher Basidiomycetes, including many taxa of the family Agaricaceae is contradictory. So it is necessary to examine the present criteria of classification and to look for new objective and reliable ones which permit to determine the leading characteristic features to differentiate the ranks of taxa.

We have also paid attention to seek for new additional specific and generic criteria using mathematical methods for the critical systematization of taxa belonging to the family Agaricaceae (WASSER, 1980). Spores and basidia as generative organs of fungi are least of all influenced by the environment, so we pointed out (BERGER & WASSER, 1976; WASSER & BERGER, 1980) that the investigation of their characteristic features is of great taxonomic value. Some authors (CORNER, 1948, 1950, 1968, 1972; MALENÇON, 1958; PERREAU-BERTRAND, 1967; BERGER & WASSER, 1976; WASSER, 1980; WASSER & BERGER, 1980) have shown that the parameters of spores, sterigmata, and basidia can be expressed by several constants common for the individuals of the same species or of the same genus. If we accurately determine such constants we shall be able to use them as reliable criteria for the species and perhaps also for the genera of higher Basidiomycetes.

We have already discussed in our papers (BERGER& WASSER, 1976; WASSER, 1980; WASSER & BERGER, 1980) some geometrical relationships between the parameters of an "ideal" basidia; we have derived several formulae which permit to calculate these parameters and proposed analytical, graphical, and nomographical methods for such calculations.

This paper deals with the practical use of our methods and examination of our formulae (and their accuracy) proposed for "ideal" basidia.

We have studied 17 species and intraspecific taxa (many of them are critical) from five genera (*Agaricus, Cystoderma, Macrolepiota, Leucoagaricus, Leucocoprinus*) of the family Agaricaceae:

1 *Cystoderma granulosum* (BATSCH: FR.) FAYOD
2 **Agaricus tabularis* PECK
3 **Leucoagaricus carneifolius* (GILL.) S. WASSER
4 **Agaricus benesii* (PIL.) PIL.
5 **Leucoagaricus leucothitus* (VITT.) S. WASSER
6 **Agaricus xanthodermus* GEN. var. *lepiotoides* R. MRE.
7 **Agaricus deylii* PIL.
8 *Macrolepiota excoriata* (SCHAEFFER: FR.) S. WASSER
9 **Agaricus romagnesii* S. WASSER
10 **Leucocoprinus pilatianus* (DEMOULIN) S. WASSER
11 **Agaricus bresadolianus* BOHUS
12 **A. meleagris* (J. SCHAEFFER) IMBACH
13 **A. xanthodermus* var. *xanthodermus* GEN.
14 **Cystoderma superbum* HUIJSMAN em. HUIJSMAN
15 **Agaricus cupreobrunneus* (J. SCHAEFFER & STEER) PIL.
16 **Agaricus moelleri* S. WASSER
17 **Agaricus porphyrocephalus* MOELL.

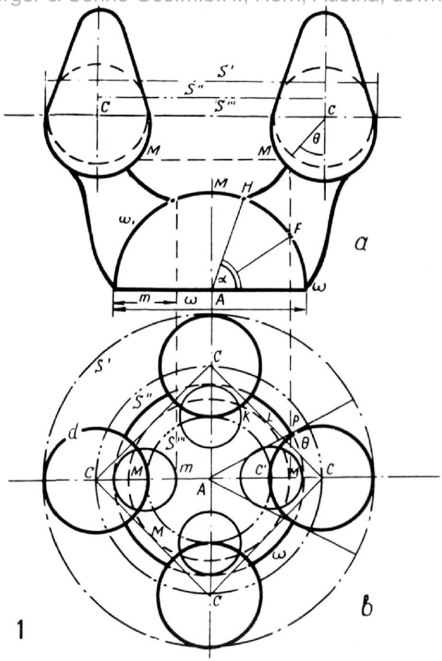
To determine each parameter, we have measured 100 spore-bearing basidia (side-view and end-view) of 17 taxa of Agaricaceae under the Amplival microscope (German Democratic Republic) using dry and immersed preparations. We have obtained the mean values of the following parameters:

d – the outline of the spore (the circle with diameter d);
S′ – the outer circle (the circle with the diameter S′, embracing the spores outlines, i.e., externally tangential to them);
ω – the outline of the basidium (the circle with the diameter ω) (see Fig. 1).

It should be noted that any two parameters mentioned above are sufficient for the mathematical determination of basidia, sterigmata, and spores parameters from our formulae (BERGER & WASSER, 1967). In this paper we use, for the following calculations, two parameters – d and S′ – as initial data. The other parameters measured under the microscope (ω, for example) are applicable for the additional examination of the proposed formulae and calculations (CORNER, 1972; BERGER & WASSER, 1976; WASSER, 1980; WASSER & BERGER, 1980).

1. Mathematical systematization of Agaricaceae taxa based on numerical criteria.

Before mathematical determination of basidia, sterigmata and spores parameters for 17 investigated taxa are to be distributed



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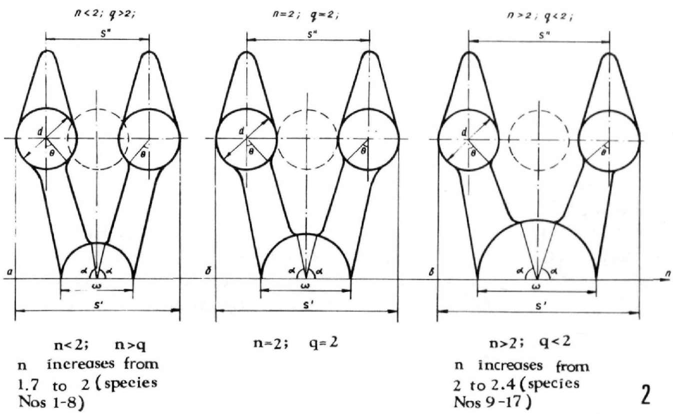

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Fig. 1. Geometry of the basidium: a = side-view; b = end-view. – Fig. 2. Schematic drawing of relative changes for basidial parameters (S′, S″, θ, α) according to the n factor increasing.

according to a certain numerical criterion we have proposed (Berger & Wasser, 1976; Wasser, 1980) a factor (n) reflecting the relationship between S″ (the central circle with the diameter S″ passing through the centers of the outlines of the spores) and d (the outline of the spore), i. e. $n = \frac{S''}{d}$; but since S″ = S′−d, then $n = \frac{S'-d}{d}$. From these data we calculate the n value for all the 17 investigates Agariacaceae taxa (see Table 1*).

Table 1. Initial data (d, S′) and factors (n, q) for calculations in taxa of Agaricaceae investigated

Ordinal number of taxa	Initial values (measured under the microscope), µm		Factors for calculation (n, q)	
	d	S′	$n = \frac{S'-d}{d}$	$q = \frac{n}{n+1}$
1	2,7	7.6	1.81	2.235
2	5.5	15.8	1.87	2.149
3	5.2	15.0	1.88	2.136
4	4.5	13.1	1.91	2.098
5	6.5	19.1	1.94	2.064
6	4.0	11.8	1.95	2.053
7	5.0	14.8	1.96	2.042
8	8.5	25.3	1.98	2.024
9	4.3	13.1	2.05	1.952
10	3.7	11.5	2.108	1.903
11	4.4	13.7	2.113	1.898
12	3.5	10.9	2.114	1.897
13	4.1	12.8	2.12	1.893
14	3.1	9.9	2.19	1.840
15	4.7	15.2	2.23	1.813
16	3.0	9.8	2.27	1.787
17	3.8	12.7	2.34	1.746

The factor n is directly proportional to S″ and inversely proportional to their diameter; n is an important marker of a relative spore size and spore distribution on the basidium: it reflects the "spore density" on basidia, i.e. when n value is higher the spores are packed more "loosely". It should be noted that when n ⩾ 2 (species Nos 9−17, Table 1) the third spore can be placed between two spores (Fig.

* Notes concerning the taxa with asterisk are given in our previous paper (Wasser, 1980).

* The taxa of the Agaricaceae are placed in Table 1 according to increasing n values. In our Tables and in the text each taxon has the same ordinal number, so we do not give the names of fungi in the text (cf. p. 294).

2). If $n<2$ (Nos. 1–8, Table 1) it is impossible, since the spore diameter is longer than the "void" space over the basidium. We can easily examine this conclusion comparing d and S''' – the inner circle (the circle with the diameter S''' limiting the "dead space" over the basidium, i. e. the circle internally tangential to the spore outlines) – Tables 1 and 2: $S''' < d$ for Nos. 1–8, $S''' > d$ for Nos. 9–17.

We have also proposed the q-factor = relation between external circle and basidium shape diameters, i. e. $q = \frac{S'}{\omega}$. According to the proposed initial data S' and d, $q = \frac{S' - d}{S' - 2d}$, but $S'' = S' - d$; $S''' = S'' - d$, consequently, $q = \frac{S'}{\omega} = \frac{S'}{S'''}$. From this formula we calculate q for all the investigated taxa of the Agaricaceae (Table 1).

It should be pointed out that the mathematical systematization of Agaricaceae taxa on the basis of this factor is the same as with n factor (no change in taxa distribution is observed). This conclusion could be drawn a priori because according to the previous data (Berger & Wasser, 1976; Wasser, 1980) q is a function of

$$n\left(q = \frac{n}{n+1}\right).$$

2. Calculations of Agaricaceae basidial linear parameters.

Table 2 presents the values S'', S''', ω, M, m calculated from initial S' and d values (see Table 1). These parameters have been derived from our formulae (Berger & Wasser, 1976):

$$S'' = S' - d,\ S'' = d \cdot n,\ S'' = d \cdot \frac{q}{q-1};$$

$$S''' = S'' - d,\ S'' = d\,(n-1),\ S''' = \frac{d}{d-1};$$

$$\omega = \frac{S' \cdot S'''}{S''},\ \omega = d \cdot \frac{n^2 - 4}{n},\ \omega = d \cdot \frac{q-1}{q\,(q-1)};$$

$$M = d\,(n-1) \cdot \sqrt{\frac{n+1}{n}},\ M = \frac{d}{q-1} \cdot \sqrt{\frac{2q-1}{q}};$$

$$m = d \cdot \frac{n-1}{n},\ m = \frac{d}{q}.$$

Comparing ω values determined from these formulae and the measured parameters we conclude that the results agree with each other.

Table 2. Linear basidial parameters for taxa of the Agaricaceae investigated (μm).

Ordinal number of taxa	Parameters					
	S″ Mean	S‴ Mean	ω Mean		M	m
	Calculated from formulae	Calculated from formulae	Calculated from formulae	Measured	Mean	Mean
1	4.894	2.194	3.41	3.8	2.728	1.2102
2	10.293	4.793	7.35	6.3	5.926	2.5609
3	9.792	4.592	7.00	7.3	5.676	2.4344
4	8.597	4.097	6.24	6.4	5.031	2.1446
5	12.604	6.104	9.248	9.3	7.521	3.1481
6	7.799	3.796	5.75	6.0	4.656	1.9487
7	9.801	4.800	7.247	7.9	5.901	2.4489
8	16.810	8.310	12.500	12.9	10.221	2.2032
9	8.805	4.500	6.700	6.5	5.505	2.2000
10	7.790	4.100	6.000	6.3	4.986	1.9448
11	9.301	4.900	7.200	7.3	5.942	2.3182
12	7.400	3.900	5.730	5.60	4.727	1.8446
13	8.696	4.596	6.750	6.7	5.568	2.1669
14	6.790	3.693	5.380	5.5	4.457	1.6856
15	10.490	5.793	8.370	8.2	5.973	2.5944
16	7.400	3.900	5.490	5.6	4.727	1.6785
17	8.900	5.100	7.277	7.4	6.095	2.1768

The error for ω parameter is norm-preserving; its mean value is 4.26%, and the highest absolute error is 1.05 μm (No. 2, Table 2).

It should be noted that our calculations based on these formulae also permit determining such basidial parameters as diameters M (the sterigmata circle; the circle with the diameter M passing through the sterigma tips) and m (sterigmata patch; the circle of diameter which is a projection of the sterigmata base); it is impossible to observe and to measure these parameters.

3. Calculation of basidial angular parameters.

No experimental technique permits measuring angular basidial parameters, so it is very difficult to determine these constants including δ (the angle between the axis AC (Fig. 1) and tangential to the spore outline), θ (a spore angle corresponding to the spore arc on the sterigma), and α (a central angle corresponding to the sterigma arc upon basidium).

To determine these parameters, it is necessary to calculate their trigonometric functions from the formulae:

$$\sin\delta = \frac{d}{S''}, \ \sin\delta = \frac{1}{n}, \ \sin\delta = \frac{q-1}{q};$$

$$\sin \theta = \frac{S'' - M}{d}, \sin \theta = n - (n - 1) \cdot \sqrt{\frac{n+1}{n}},$$

$$\sin \theta = \frac{q - \sqrt{\frac{2q-1}{q}}}{q - 1};$$

$$\cos \alpha = \frac{S''' - m}{\omega}, \cos \alpha = \frac{n - 1}{n + 1}, \cos \alpha = \frac{1}{2q - 1}.$$

Then we determine the mean angular parameters and corresponding values of angles (Table 3) from the trigonometric function Tables. It is evident that if n increases (or q decreases), the values of δ, θ, and α angles decrease correspondingly. But the difference between the highest and the lowest values of angles for 17 investigated taxa of Agaricaceae is negligible (8° or less).

We tried to show the relative changes of investigated parameters more clearly; the "ideal" basidium containing spores of the same diameter(d) and different n and q values are graphically presented in Fig. 2.

4. Tabular-nomographic method for determination of Agaricaceae basidial parameters.

The paramenters of agaricoid basidia (n and d values) are given in Table 4. The data of this Table simplify the calculation of other necessary parameters.

For example, the angular parameters sin δ, δ, sin θ, θ, cos α and q factor can be found in this Table without additional calculations according to the corresponding n value.

Linear basidial parameters S′, S″, S‴, ω, M, m can be calculated as a result of multiplication of d values (spore diameter) by corresponding factors. Their values, derived from formulae (1–84), have been published in our previous papers (Berger & Wasser, 1976; Wasser & Berger, 1980). In 17 investigated Agaricaceae taxa the value of n factor is in limits of 1.8–2.5; we have composed a detailed parametric Table (4) for different n values (1.5–1.8 with a spacing 0.1 and 1.8–2.0 with a spacing 0.01) applicable for the determination of the majority of the Agaricaceae taxa found in the USSR.

It is easy to find the localization of 17 investigated taxa in Table 4 and to determine the values of all basidial parameters for each taxon.

The identical results can be also obtained using nomographs. A nomograph given in Fig. 3 permits to determine:

1) α, θ and δ values (from the scale of angles) and the q value for the corresponding n factor value;

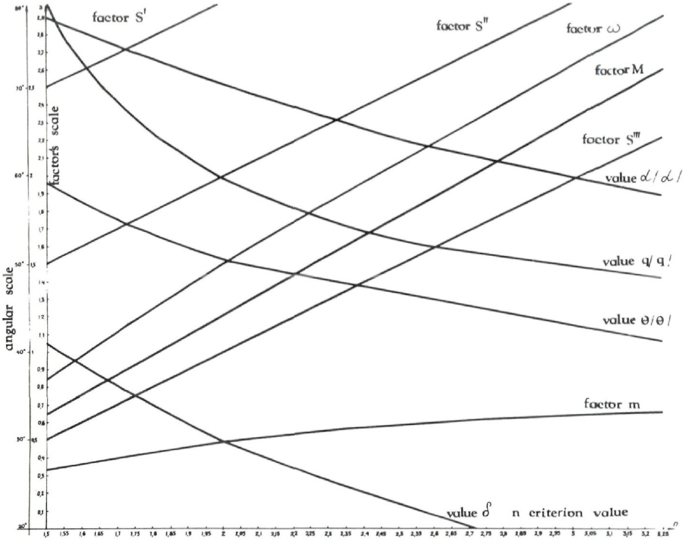
Table 3. Angular basidial parameters for taxa of Agaricaceae investigated

Ordinal number of taxa	Parameter and number of formula used for calculation (WASSER & BERGER, 1980)													
	sin δ (S) (68)	sin δ (69)	sin δ Mean	δ	sin θ (17)	sin θ (72)	sin θ (73)	sin θ Mean	θ	cos α (9)	cos α (70)	cos α (71)	cos α Mean	α
1	0.5510	0.5519	0.5514	33°28′	0,803	0,802	0,800	0,8020	53°18′	0.2885	0.2883	0.2882	0.2883	73°13′
2	0.5340	0.5342	0.5341	32°17′	0,800	0,794	0,793	0,7956	52°42′	0.3036	0.3031	0.3032	0.3033	72°21′
3	0.5306	0.5305	0.5306	32°3′	0,792	0,791	0,790	0,7910	52°17′	0.3078	0.3056	0.3056	0.3063	72°11′
4	0.5332	0.5234	0.5233	31°33′	0,791	0,788	0,788	0,7880	52°	0.3128	0.3127	0.3129	0.3128	71°47′
5	0.5160	0.5157	0.5168	31°9′	0,783	0,784	0,785	0,7840	51°39′	0.3196	0.3197	0.3197	0.3197	71°20′
6	0.5128	0.5129	0.5128	30°51′	0,782	0,782	0,784	0,7830	51°33′	0.3213	0.3220	0.3219	0.3117	71°15′
7	0.5102	0.5100	0.5102	30°40′	0,780	0,780	0,780	0,7800	51°17′	0.3244	0.3243	0.3243	0.3243	71°3′
8	0.5060	0.5059	0.5060	30°24′	0,776	0,776	0,766	0,7760	50°54′	0.3285	0.3288	0.3281	0.3285	70°49′
9	0.4886	0.4885	0.4886	29°9′	0,767	0,768	0,769	0,7685	50°13′	0.3433	0.3442	0.3444	0.3439	69°53′
10	0.4744	0.4737	0.4741	28°18′	0,766	0,766	0,764	0,7650	49°55′	0.3548	0.3545	0.3544	0.3545	69°30′
11	0.4731	0.4730	0.4731	28°15′	0,764	0,765	0,764	0,7645	49°51′	0.3565	0.3565	0.3564	0.3565	69°43′
12	0.4729	0.4729	0.4729	28°13′	0,761	0,764	0,764	0,7630	49°45′	0.3580	0.3578	0.3577	0.3578	69°2′
13	0.4713	0.4714	0.4713	28°7′	0,755	0,763	0,762	0,7625	40°40′	0.3590	0.3590	0.3589	0.3592	68°57′
14	0.4560	0.4565	0.4562	27°9′	0,755	0,752	0,752	0,7520	48°45′	0.3731	0.3730	0.3731	0.3731	68°5′
15	0.4476	0.4475	0.4476	26°35′	0,751	0,749	0,748	0,7485	48°25′	0.3811	0.3808	0.3808	0.3809	67°37′
16	0.4412	0.4400	0.4409	26°9′	0,771	0,746	0,746	0,7460	48°15′	0.3880	0.3884	0.3885	0.3883	67°8′
17	0.4267	0.4269	0.4268	25°15′	0,737	0,737	0,737	0,7370	47°27′	0.4017	0.4012	0.4013	0.4014	66°19′

Note: The values of sin δ calculated from our formulae (5) and (68) agree completely.

Table 4. Tabular method for the determination of basidial parameters in Agaricaceae

Ordinal number of taxa	Factors			Basidial				parameters				
	n	q	d	S′	S″	S‴		M	m	sin δ	sin θ	cos α
	1,5	3,0	–	2,5d	1,5d	0,5d	0,8333d	0,645d	0,3333d	0,6666	0,855	0,2
	1,6	2,666	–	2,6d	1,6d	0,6d	0,975d	0,765d	0,375d	0,6255	0,835	0,2308
	1,7	2,4285	–	2,7d	1,7d	0,7d	1,118d	0,881d	0,4118d	0,5882	0,819	0,2593
	1,8	2,25	–	2,8d	1,8d	0,8d	1,244d	0,998d	0,4444d	0,5555	0,802	0,2857
1	1,81	2,235	2,7	2,81d	1,81d	0,81d	1,255d	1,009d	0,4475d	0,5519	0,801	0,2883
				7,587	4,887	2,187	3,395	2,724	1,2082			
	1,82	2,2195	–	2,82d	1,82d	0,82d	1,2705d	1,02d	0,4505d	0,5495	0,8	0,2908
	1,83	2,2048	–	2,83d	1,83d	0,83d	1,2836d	1,03d	0,4536d	0,5464	0,8	0,2933
	1,84	2,1905	–	2,84d	1,84d	0,84d	1,2965d	1,04d	0,4565d	0,5435	0,8	0,2958
	1,85	2,1765	–	2,85d	1,85d	0,85d	1,3095d	1,05d	0,4595d	0,5405	0,8	0,2982
	1,86	2,1628	–	2,86d	1,86d	0,86d	1,3224d	1,06d	0,4624d	0,5376	0,8	0,3007
2	1,87	2,149	–	2,87d	1,87d	0,87d	1,3352d	1,078d	0,4652d	0,5343	0,8	0,3031
				15,785	10,285	4,785	7,344	5,929	2,5586			
3	1,88	2,136	5,2	2,88d	1,88d	0,88d	1,348d	1,089d	0,4681d	0,5310	0,7920	0,3060
				14,976	9,776	4,576	7,0	5,663	2,4344			
	1,89	2,1236	–	2,89d	1,89d	0,89d	1,3609d	1,1d	0,4709d	0,5291	0,7910	0,3079
	1,9	2,1111	–	2,9d	1,9d	0,9d	1,3737d	1,11d	0,4737d	0,5263	0,7905	0,3103
4	1,91	2,098	4,5	2,91d	1,91d	0,91d	1,3864d	1,123d	0,4764d	0,5235	0,79	0,3127
				13,095	8,595	4,095	6,24	5,053	2,1438			
	1,92	2,0869	–	2,92d	1,92d	0,92d	1,3992d	1,134d	0,4792d	0,5208	0,787	0,3151
	1,93	2,0753	–	2,93d	1,93d	0,93d	1,4119d	1,146d	0,4819d	0,5181	0,786	0,3174
5	1,94	2,064	6,5	2,94d	1,94d	0,94d	1,4245d	1,157d	0,4845d	0,5160	0,784	0,3197
				19,11	12,61	6,11	9,259	7,521	3,1499			
6	1,95	2,053	4,0	2,95d	1,95d	0,95d	1,4372d	1,168d	0,4872d	0,5128	0,783	0,3220
				11,8	7,8	3,8	5,75	4,672	1,9488			
7	1,96	2,042	5,0	2,96d	1,96d	0,96d	1,4498d	1,179d	0,4898d	0,5102	0,782	0,3243
				14,8	9,800	4,8	7,249	5,895	2,4490			
8	1,97	2,0309	–	2,97d	1,97d	0,97d	1,4624d	1,191d	0,4924d	0,5076	0,781	0,3266
	1,98	2,024	8,5	2,98d	1,98d	0,98d	1,4749d	1,202d	0,4949d	0,5059	0,779	0,3288
				25,33	16,83	8,33	12,53	10,217	4,2067			
9	2,0	2,0	–	3d	2d	1d	1,5d	1,225d	0,5d	0,5	0,775	0,3333
				3,05d	2,05d	1,05d	1,5622d	1,281d	0,5122d			
	2,05	1,952	4,3	13,115	8,815	4,515	6,6446	5,508	2,2	9,4878	0,769	0,3443



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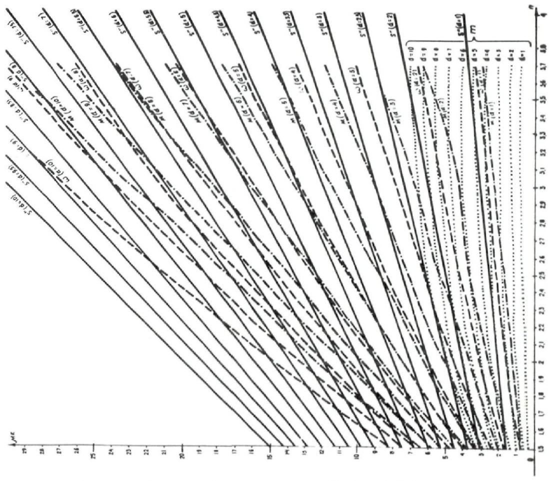

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Fig. 3. Nomograph permitting to determine angular basidial parameters and factors for calculation of linerar parameters m, M, S′, S″, S‴ and ω. – Fig. 4. Nomograph for the determination of linear basidial parameters from given n and d values.

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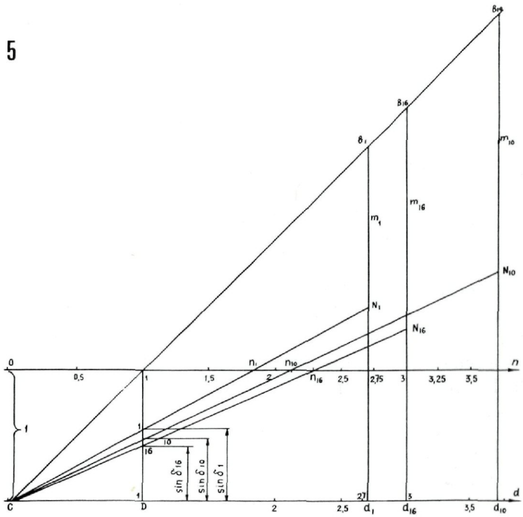

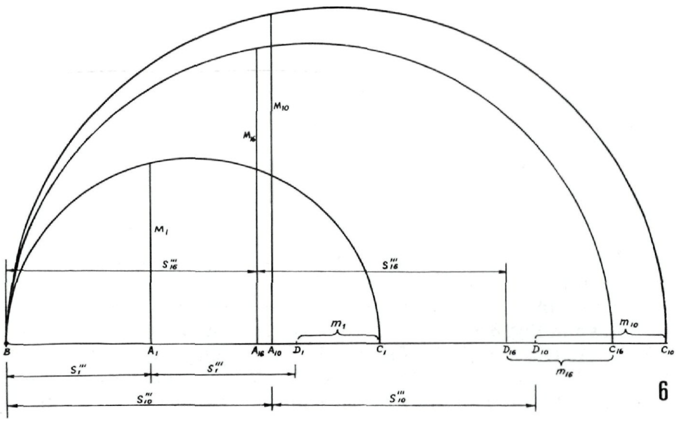

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Fig. 5. Graph used for determination of m and sin δ values for the species Nos. 1, 10, 16. – Fig. 6. Graph used for determination of ω and M values for the species Nos. 1, 10, 16.

2) factors for the determination of S′, S″, S‴, ω, M, m values for d function. These values can be calculated by means of multiplicating the factors by corresponding d values (the outline of the spore).

The nomograph presented in Fig. 4 permits direct determination of linear basidial parameters (S″, ω, m, M) without previous calculation of factor values. The values of parameters mentioned above are seen on the vertical numeral scale crossing the curve corresponding to a given d value (different n values are presented on the abscissa).

5. Graphical method for determination of basidial parameters in Agaricaceae.

Our previous publications (BERGER & WASSER, 1976; WASSER, 1980; WASSER & BERGER, 1980) deal with graphical methods proposed for the determination of "ideal" basidial parameters. In this paper we examine their applicability for the calculation of some Agaricaceae basidial parameters for some taxa (Nos. 1,10,16).

Fig. 5 illustrates a method of determination of two parameters – m and sin δ, $\mu_e = 0.02$. Their corresponding values are set upon the *on* and *cd* scales ($n_1 = 1.81$; $n_{10} = 2.1$, $n_{16} = 2.27$ and $d_1 = 2.7$, $d_{10} = 3.7$, $d_{16} = 3$), the lines cn_1, cn_{10}, cn_{16}, and d_1B_1, $d_{10}B_{10}$, $d_{16}B_{16}$ are drafted and their intersection points are shown. The following results are obtained from these data:

$$m_1 = \overline{B_1N_1} \cdot \mu_e = 62 \cdot 0.02 = 1.24\ (1.21\ \mu m);$$
$$m_{10} = \overline{B_{10}N_{10}} \cdot \mu_e = 98 \cdot 0.02 = 1.96\ (1.945\ \mu m);$$
$$m_{16} = \overline{B_{16}N_{16}} \cdot \mu_e = 85 \cdot 0.02 = 1.7\ (1.6785\ \mu m);$$
$$\sin \delta_1 = \overline{1 - D} \cdot \mu_e = 28 \cdot 0.02 = 0.56\ (0.5519\ \mu m);$$
$$\sin \delta_{10} = \overline{10 - D} \cdot \mu_e = 24 \cdot 0.02 = 0.48\ (0.47\ \mu m);$$
$$\sin \delta_{16} = \overline{16 - D} \cdot \mu_e = 22 \cdot 0.02 = 0.44\ (0.4405\ \mu m).$$

The data in parentheses obtained by tabular method are presented for comparison.

The graphical methods which permit to obtain ω and M values are shown in Fig. 6 ($\mu_e = 0.04$). The cuts $BA_1 = A_1D_1 = S_1''' = 2.19\ \mu m$, $BA_{10} = A_{10}D_{10} = S_{10}''' = 4.09\ \mu m$, $BA_{16} = A_{16}D_{16} = S_{16}''' = 3.81\ \mu m$. The cuts $D_1C_1 = m_1 = 1.24\ \mu m$, $D_{10}C_{10} = m_{10} = 1.96\ \mu m$ are "added" to them relatively; the circles are then drafted with BC cuts as diameters. As a result we have:

$$\omega_1 = S_1''' + m_1 = \overline{A_1C_1} \cdot \mu_e = 86 \cdot 0.04 = 3.44\ (3.395\ \mu m);$$
$$\omega_{10} = S_{10}''' + m_{10} = \overline{A_{10}C_{10}} \cdot \mu_e = 150 \cdot 0.04 = 6\ (6\ \mu m);$$

$\omega_{16} = S_{16}''' + m_{16} = \overline{A_{16}C_{16}} \cdot \mu_e = 134 \cdot 0.04 = 5.36$ (5.488 µm);
$M_1 = \overline{A_1E_1} \cdot \mu_e = 69 \cdot 0.04 = 2.76$ (2.724 µm);
$M_{10} = \overline{A_{10}E_{10}} \cdot \mu_e = 122 \cdot 0.04 = 4.88$ (4.976 µm);
$M_{16} = \overline{A_{16}E_{16}} \cdot \mu_e = 112 \cdot 0.04 = 4.48$ (4.575 µm);

The data in parentheses (ω and M) are obtained by the tabular method.

6. Examination of basidial properties in some Agaricaceae.

Having analyzed the derived formulae we could describe (Berger & Wasser, 1976) certain new properties that reflect some dependencies between basidial parameters. For instance, from the formula

$$q = \frac{S'}{\omega} = \frac{S''}{S'''} = \frac{d}{m}$$

we have the following:

1. The ratio of the outer circle and basidium, outline diameters is equal to the ratio of the central and inner circles diameters.
2. The ratio of the outer circle and basidium outline diameters is equal to the ratio of spore outline and sterigmatic patch diameters.
3. The ratio of the central and inner circles diameters is equal to the ratio of the spore outline and sterigmatic patch diameters.

By analogy, the formula $n = \frac{S''}{d} = \frac{S'''}{m}$ suggests that the ratio of the central circle and spore oultline diameters is equal to the ratio of the outer circle and sterigmatic patch diameters (4). From the formula $\frac{S'}{d} = \frac{\omega}{m}$ it is evident that the ratio of the outer circle and spore outline diameters is the same as the ratio of the basidium outline and sterigmatic patch diameters (5).

We have examined the presence of these properties in Agaricaceae belonging to the investigated taxa. We have found the ratio between the parameters mentioned above for each taxon and compared them. For example, the value of S'/ω for the species No. 1 is 2.2273. So $S''/S''' = 2.23$, $d/m = 2.2314$. Consequently, it is correct to declare that these ratios are equal (the difference in the third decimal place in negligible). In the same way we have examined the properties of all investigated Agaricaceae taxa (see Table 5).

Our data suggest the study of geometrical dependence in basidia to be perspective; several new parameters described above are of specific and probably generic taxonomic value. The dependences

Table 5. Examination of the basidial characters for the investigated Agaricaceae fungi

Ordinal Number of taxa	Characters							
	1,2,3				4		5	
	$q = \frac{n}{n-1}$	$q = \frac{S'}{\omega}$	$q = \frac{S''}{S'''}$	$q = \frac{d}{m}$	$n = \frac{S''}{d}$	$n = \frac{S'''}{m}$	$\frac{S'}{d}$	$\frac{\omega}{m}$
1	2.235	2.2273	2.2300	2.2314	1.8100	1.8180	2.8148	2.8177
2	2.149	2.1458	2.1458	2.1480	1.8700	1.8750	2.8727	2.8701
3	2.136	2.1304	2.1300	2.1310	1.8800	1.8850	2.8846	2.8755
4	2.098	2.0976	2.0980	2.0980	1.9100	1.9123	2.9111	2.9096
5	2.064	2.0656	2.0650	2.0650	1.9400	1.9380	2.9384	2.9376
6	2.053	2.0526	2.0530	2.0526	1.9500	1.9500	2.9500	2.9507
7	2.042	2.0417	2.0410	2.0408	1.9600	1.9600	2.9600	2.9593
8	2.024	2.0241	2.0240	2.0240	1.9800	1.9760	2.9764	2.9739
9	1.952	1.9555	1.9550	1.9545	2.0500	2.0450	3.0465	3.0455
10	1.903	1.9024	1.9000	1.9023	2.1080	2.1079	3.1081	3.0850
11	1.898	1.8980	1.8970	1.8965	2.1136	2.1138	3.1136	3.1058
12	1.897	1.8974	1.8974	1.8974	2.1143	2.1143	3.1143	3.1064
13	1.893	1.8910	1.8920	1.8920	2.1200	2.1230	3.1219	3.1150
14	1.840	1.8378	1.8400	1.8410	2.1900	2.1945	3.1935	3.1917
15	1.813	1.8103	1.8103	1.8147	2.2300	2.2300	3.2340	3.2262
16	1.787	1.7894	1.7870	1.7860	2.2700	2.2660	3.2667	3.2710
17	1.746	1.7451	1.7451	1.7455	2.3400	2.2390	3.3421	3.3429

found for "ideal" basidia have been examined for 17 Agaricaceae taxa; they proved to be useful as additional taxonomic criteria. For example, the related species Nos. 12 and 16, difficult to separate due to their morphological resemblance, are clearly differentiated from their S′, S″, and S‴ parameters; the taxa Nos. 13 and 6 are easily identified taking into consideration their S′, S″, ω, M, δ, θ and α values; the taxa Nos. 15 and 17 are closely related critical species (WASSER, 1980) perfectly differentiated by their S′, S″ and ω parameters.

Our formulae also permit to calculate basidial parameters such as M, m, δ, θ, α which are hardly observed and can not be measured. All parameters of Agaricaceae basidia are determined quickly and easily by tabular-monographic and graphical methods.

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